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PANEL SOUND ABSORBERS

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A theory is developed that predicts the variation with frequency of the acoustic impedance of a panel sound absorber. This theory takes into account the mass, stiffness and internal damping of the panel; the stiffness of the cavity; and also the effect of introducing a porous material into the cavity. The equation of motion of the panel is derived and is solved for the first four natural modes of vibration. The resulting impedances are plotted on Argand diagrams. A normal incidence standing wave interferometer was used to determine, experimentally, the acoustic impedance of a panel absorber with various amounts of damping. Good agreement is shown between the theoretical and experimentally observed values.

1. INTRODUCTION

The acoustic behaviour of membrane sound absorbers and their use in broadcasting studios, concert halls, etc., has been well described [1]. In brief, they are low frequency absorbers and consist essentially of rectangular boxes with sturdy sides and backs and limp lightweight fronts. Fibrous material is placed inside the box to provide damping of the induced air movement. The dimensions of the box are such that no standing waves occur in it at the frequencies to be absorbed, and hence the enclosed air acts as a simple spring. The mass-spring system thus formed is a resonant one and absorption occurs at and around the resonant frequency.

The frequency of maximum absorption of such a system has been successfully predicted but little information is available on the variation of its acoustic impedance with frequency, especially for the case where the membrane is sufficiently stiff to influence the absorption mechanism. A full understanding of the action of this type of absorber was found necessary as a result of efforts to manufacture miniature absorbers for use in scale models. All attempts to construct small absorbers with limp membranes failed, thus necessitating the use of absorbers with stiff membranes.

However, the use of a stiff membrane would give a full-size absorber certain advantages. The construction is simpler as no protective covering is required for the membrane; material can be fixed to the outside of the panel to provide either high frequency absorption or simply a decorative finish; and a wider range of materials can be used for the panel.

2. THEORY

2.1. EQUATION OF MOTION OF THE PANEL

It is assumed that the panel is sufficiently rigid to act as a plate, is clamped along all four edges, and that no static stresses exist in it. It is also assumed that the plate is thin compared with the other dimensions and of uniform thickness, thereby eliminating any shear effects; that stresses in the middle plane of the plate due to the vibration are negligible, thereby eliminating any membrane action; that it consists of a perfectly elastic, homogeneous material; and that only transverse displacements occur.

Let the plane of the panel be defined by the x and y directions and the direction perpendicular to the panel by z, the displacement at any point on the panel at any time in this direction being w. Then if W is the amplitude of the displacement at the point under consideration; for harmonic motion

$$w = W e^{j\omega t}. \tag{1}$$

Consequently the maximum displacement as a function of time, ϕ, will be, if Φ is the maximum displacement amplitude of the panel,

$$\phi = \Phi e^{j\omega t}. \tag{2}$$

The displacement at any point on the panel is related to the maximum displacement by the equation

$$w = \phi . f(x) . g(y) \tag{3}$$

and depends on the mode shape assumed by the panel while vibrating. The functions chosen for $f(x)$ and $g(y)$ are those for a beam clamped at both ends. However, as even-numbered modes, i.e. modes having an odd number of nodal lines, cannot be excited by the action of a uniformly distributed pressure only odd-numbered modes need to be considered. Hence, from beam theory [2],

$$f(x) = \cos\gamma\left(\frac{x}{a} - \frac{1}{2}\right) + k\cosh\gamma\left(\frac{x}{a} - \frac{1}{2}\right), \tag{4}$$

$$g(y) = \cos\epsilon\left(\frac{y}{b} - \frac{1}{2}\right) + c\cosh\epsilon\left(\frac{y}{b} - \frac{1}{2}\right), \tag{5}$$

where

$$k = \sin\frac{\gamma}{2}\Big/\sinh\frac{\gamma}{2}, \tag{6a}$$

$$c = \sin\frac{\epsilon}{2}\Big/\sinh\frac{\epsilon}{2}, \tag{6b}$$

$$\tan\frac{\gamma}{2} + \tanh\frac{\gamma}{2} = 0, \tag{7a}$$

$$\tan\frac{\epsilon}{2} + \tanh\frac{\epsilon}{2} = 0. \tag{7b}$$

Under these conditions the displacement ϕ occurs at the centre of the panel, and several modes of vibration are possible. Consequently equation (3) becomes

$$w = \sum_m \sum_n \phi_{mn} f_m(x) \cdot g_n(y) \tag{8}$$

where the subscripts denote in which modes the variables are involved. The subscript m is used to denote modes in the x direction and n to denote modes in the y direction. For ease of writing, however, the subscripts on γ and k (m), and ϵ and c (n), will be omitted. The displacement w is thus

$$w = \sum_m \sum_n \phi_{mn}\left[\cos\gamma\left(\frac{x}{a} - \frac{1}{2}\right) + k\cosh\gamma\left(\frac{x}{a} - \frac{1}{2}\right)\right].\left[\cos\epsilon\left(\frac{y}{b} - \frac{1}{2}\right) + c\cosh\epsilon\left(\frac{y}{b} - \frac{1}{2}\right)\right]. \tag{9}$$

It is now possible to calculate for any time; (a) the total energy of the system, which is composed of the potential and kinetic energies of the panel and the potential energy of the enclosed air; (b) the rate at which energy is dissipated by damping forces acting on the system. Then by using the principle of virtual work the equation of motion of the panel can be obtained.

2.1.1. *Potential energy of the panel*

The potential energy, U_p, of a vibrating plate is, if $w = 0$ at all four edges [3, 4],

$$U_p = \frac{D}{2}\int_0^a\int_0^b \left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 + 2\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)\mathrm{d}x\,\mathrm{d}y \tag{10}$$

where

$$D = \frac{Ed_1^3}{12(1-\sigma^2)}. \tag{11}$$

The partial differentials are obtained from equation (8) and hence

$$U_p = \frac{D}{2}\int_0^a\int_0^b \left[\sum_m\sum_n \phi_{mn}.g(y).\frac{\partial^2 f(x)}{\partial x^2}\right]^2 + \left[\sum_m\sum_n \phi_{mn}.f(x).\frac{\partial^2 g(y)}{\partial y^2}\right]^2 + 2\left[\sum_m\sum_n \phi_{mn}.\frac{\partial f(x)}{\partial x}.\frac{\partial g(y)}{\partial y}\right]^2.\mathrm{d}x.\mathrm{d}y. \tag{12}$$

It can be shown that the integrals of the cross terms formed by squaring the brackets are all zero. Also the solution of equation (7) approximates to

$$\frac{\gamma}{2} = \frac{\epsilon}{2} = (\eta - \tfrac{1}{4})\pi \tag{13}$$

where η is an integer. This approximation is very good for $\eta \geqslant 2$ but a 2% error is introduced into $\gamma/2$ and $\epsilon/2$ when taking $\eta = 1$. Carrying out the integration,

$$U_p = \frac{D}{2}\sum_m\sum_n \phi_{mn}^2\left[\left(\frac{\gamma^4}{a^4}+\frac{\epsilon^4}{b^4}\right)\frac{ab}{4}(1+c^2)(1+k^2) + \frac{\epsilon^2\gamma^2}{2ab}(1-c^2)(1-k^2) + \frac{\epsilon\gamma}{ab}(2-\epsilon(1-c^2)-\gamma(1-k^2))\right]. \tag{14}$$

It is now necessary to express this energy in terms of the average displacement of the panel $\bar{w}$, so that it can be used to calculate the acoustic impedance of the absorber. The average displacement per mode is given by

$$\bar{w}_{mn} = \frac{1}{ab}\int_0^a\int_0^b w_{mn}\,\mathrm{d}x\,\mathrm{d}y \tag{15}$$

whence

$$\phi_{mn}^2 = \frac{\gamma^2\epsilon^2}{64}\bar{w}_{mn}^2. \tag{16}$$

Thus, for a square plate, equation (14) becomes

$$U_p = \frac{D}{2a^2} \sum_m \sum_n B_{mn} \bar{w}_{mn}^2 \tag{17}$$

where

$$B_{mn} = \frac{\gamma^2 \epsilon^2}{256} [(\gamma^4 + \epsilon^4)(1 + c^2)(1 + k^2) + 2\gamma^2 \epsilon^2 (1 - c^2)(1 - k^2) + 4\gamma\epsilon(2 - \epsilon(1 - c^2) - \gamma(1 - k^2))]. \tag{18}$$

2.1.2. *Kinetic energy of the panel*

The kinetic energy, T_p, of a vibrating plate is [3]

$$T_p = \frac{M}{2} \int_0^a \int_0^b \left(\frac{\partial w}{\partial t}\right)^2 . \mathrm{d}x . \mathrm{d}y. \tag{19}$$

Substituting for $\partial w/\partial t$ from equation (9) and integrating,

$$T_p = \frac{Mab}{8} \sum_m \sum_n \dot{\phi}_{mn}^2 (1 + c^2)(1 + k^2). \tag{20}$$

N.B. The dot nomenclature will be used to mean the derivative with respect to time.

In this case the energy has to be expressed in terms of the average velocity $\bar{\dot{w}}$, which is readily obtainable from equation (16). The kinetic energy of a square panel is thus

$$T_p = \frac{Ma^2}{2} \sum_m \sum_n A_{mn} \bar{\dot{w}}_{mn}^2 \tag{21}$$

where

$$A_{mn} = \frac{\epsilon^2 \gamma^2}{256} (1 + c^2)(1 + k^2). \tag{22}$$

2.1.3. *Potential energy of the air in the cavity*

Because the cavity is completely sealed the enclosed air acts as a spring which stores potential energy but does not possess any kinetic energy due to its extremely small mass. If no standing waves exist in the cavity, which is the normal condition for a membrane absorber, the stiffness of the spring is constant, and the potential energy, U_a, stored is [1]

$$U_a = \frac{\gamma_0 P_0}{2v} (\delta v)^2. \tag{23}$$

Hence, for a square panel,

$$U_a = \frac{\gamma_0 P_0}{2d} a^2 \left(\sum_m \sum_n \bar{w}_{mn}\right)^2. \tag{24}$$

2.1.4. *Hysteretic damping of the panel*

Hysteretic damping of the panel is provided by internal friction in the panel material, the energy loss being represented by the area of the stress-strain loop. This energy loss, although in phase with the velocity, is more nearly proportional to the amplitude of vibration. It has been shown by other authors [5, 6] that the hysteretic damping can be included in the equation of motion as $j \times g_{mn} \times$ bending stiffness of panel, where g_{mn} is the loss factor of the panel material and varies with the mode of vibration. The bending stiffness can, therefore, be regarded as being complex, and equation (17) for the potential energy of the panel can be rewritten as

$$U_p = \frac{D}{2a^2} \sum_m \sum_n (1 + jg_{mn}) B_{mn} \bar{w}_{mn}^2. \tag{25}$$

2.1.5. *Principle of virtual work*

If the panel is caused to move through an infinitesimal displacement, $\delta\bar{w}$, then the change in energy incurred, if E_T is the total energy in the system, is $(\partial E_T/\partial\bar{w})\delta\bar{w}$, and is equal to the work done by the driving force, whether this is real or virtual, i.e.,

$$\frac{\partial T_p}{\partial \bar{w}}.\delta\bar{w} + \frac{\partial U_p}{\partial \bar{w}}.\delta\bar{w} + \frac{\partial U_a}{\partial \bar{w}}.\delta\bar{w} = F e^{j\omega t}.\delta\bar{w}. \tag{26}$$

These partial differentials are obtained from equations (21), (24) and (25), and when inserted into equation (26) give

$$MA_{mn}\ddot{\bar{w}}_{mn} + \frac{D(1+jg_{mn})}{a^4}.B_{mn}\bar{w}_{mn} + \frac{\gamma_0 P_0}{d}\sum_m \sum_n \bar{w}_{mn} = p\,e^{j\omega t}, \tag{27}$$

where

$$p = F/a^2, \tag{28}$$

and the summations are taken over all m and n consistent with the condition that only odd-numbered modes are considered.

This equation is the equation of motion of the panel for one mode only, there being one such equation for each mode. These can be solved simultaneously by the use of matrices to obtain the average velocity amplitude of the panel over all modes, V_s, i.e.

$$V_s = \sum_m \sum_n \bar{V}_{mn}. \tag{29}$$

The specific acoustic impedance of the absorber, Z_a, is then found from

$$Z_a = p/V_s, \tag{30}$$

where V_s is a complex quantity and contains information as to the relative phase of the motion of the panel compared with that of the applied pressure.

2.2. SOLUTION

Only the first four solutions of equation (27) are considered in this paper, namely those for the (1, 1), (1, 3), (3, 1) and (3, 3) modes, as modes of higher orders are of little significance.

In the case of a square plate it has been shown that when the number of half-wavelengths m and n are related by the equation

$$m - n = \pm 2, \quad \pm 4, \quad \pm 6, \ldots, \text{etc.}, \tag{31}$$

the modes (m,n) and (n,m) do not exist separately. Instead, a displacement pattern is formed from the sum or difference of the two modal patterns; in such combined modes, represented by $m/n \pm n/m$, the nodal lines are not parallel to the edges. These modes have resonant frequencies that differ slightly and lie on either side of that calculated for the (m,n) mode. However, only a very small error is introduced if this fact is ignored, and so for simplicity it has been assumed that the (1, 3) and (3, 1) modes of the panel are identical.

It follows that it is only necessary to solve three simultaneous differential equations to obtain the total velocity amplitude. These equations can be written in matrix form thus, remembering that

$$\bar{w}_{mn} = \bar{V}_{mn} e^{j\omega t}: \tag{32}$$

$$\begin{vmatrix} \frac{B_{11} D g_{11}}{\omega a^4} + j\left(\omega M A_{11} - \frac{B_{11} D}{\omega a^4} - \frac{\gamma_0 P_0}{\omega d}\right) & -j\left(\frac{2\gamma_0 P_0}{\omega d}\right) & -j\left(\frac{\gamma_0 P_0}{\omega d}\right) \\ -j\left(\frac{\gamma_0 P_0}{\omega d}\right) & \frac{B_{13} D g_{13}}{\omega a^4} + j\left(\omega M A_{13} - \frac{B_{13} D}{\omega a^4} - \frac{2\gamma_0 P_0}{\omega d}\right) & -j\left(\frac{\gamma_0 P_0}{\omega d}\right) \\ -j\left(\frac{\gamma_0 P_0}{\omega d}\right) & -j\left(\frac{2\gamma_0 P_0}{\omega d}\right) & \frac{B_{33} D g_{33}}{\omega a^4} + j\left(\omega M A_{33} - \frac{B_{33} D}{\omega a^4} - \frac{\gamma_0 P_0}{\omega d}\right) \end{vmatrix} \begin{vmatrix} \bar{V}_{11} \\ \bar{V}_{13} \\ \bar{V}_{33} \end{vmatrix} = \begin{vmatrix} p \\ p \\ p \end{vmatrix}, \tag{33}$$

By inverting the matrix, expressions can be obtained for $\bar{V}_{11}$, $\bar{V}_{13}$ and $\bar{V}_{33}$, which, when inserted into equation (30), yield

$$Z_a = R_a + jX_a = \frac{GS - NT}{S^2 + T^2} - j\left[\frac{NS + GT}{S^2 + T^2}\right], \tag{34}$$

where

$$G = LHK - LH_1 K_1 - L_1 H_1 K - L_1 K_1 H + Q_1^2(2H + 2L + K), \tag{35a}$$

$$N = H_1 L_1 K_1 - L_1 HK - LH_1 K - LHK_1 - Q_1^2(2H_1 + 2L_1 + K_1) + 4Q_1^3, \tag{35b}$$

$$S = KL - L_1 K_1 + 2HL - 2H_1 L_1 + HK - H_1 K_1 + 2Q_1(2H_1 + 2L_1 + K_1) - 6Q_1^2, \tag{35c}$$

$$T = LK_1 + KL_1 + 2HL_1 + 2H_1 L + HK_1 + KH_1 - 2Q_1(2H + 2L + K), \tag{35d}$$

$$Q_1 = -\frac{\gamma_0 P_0}{\omega d}, \tag{35e}$$

$$H_1 = \omega M A_{11} - \frac{B_{11} D}{\omega a^4} - \frac{\gamma_0 P_0}{\omega d} \qquad H = \frac{B_{11} D g_{11}}{\omega a^4}, \tag{35f}$$

$$K_1 = \omega M A_{13} - \frac{B_{13} D}{\omega a^4} - \frac{2\gamma_0 P_0}{\omega d} \qquad K = \frac{B_{13} D g_{13}}{\omega a^4}, \tag{35g}$$

$$L_1 = \omega M A_{33} - \frac{B_{33} D}{\omega a^4} - \frac{\gamma_0 P_0}{\omega d} \qquad L = \frac{B_{33} D g_{33}}{\omega a^4}. \tag{35h}$$

The constants A_{mn} and B_{mn} apply to any plate clamped under the conditions stated, and can be calculated immediately from equations (18) and (22), using equations (6), (7) and (13), i.e.

$$A_{11} = 2{\cdot}02 \qquad B_{11} = 2{\cdot}64 \times 10^3, \tag{36a}$$

$$A_{13} = 10{\cdot}8 \qquad B_{13} = 1{\cdot}89 \times 10^5, \tag{36b}$$

$$A_{31} = 10{\cdot}8 \qquad B_{31} = 1{\cdot}89 \times 10^5, \tag{36c}$$

$$A_{33} = 57{\cdot}1 \qquad B_{33} = 2{\cdot}79 \times 10^6. \tag{36d}$$

2.3. THE EFFECT OF DAMPING MATERIAL IN THE CAVITY

If a porous material such as glass fibre or flexible polyurethane foam is inserted into the cavity energy will be dissipated by the viscous oscillatory flow of air through the material. This energy loss is in phase with, and proportional to, the velocity of air through the pores and is therefore not constant throughout the material. It is thus not possible to define a simple resistance coefficient for the material, which could be included in the equation of motion. However, it is possible to calculate the effect of the impedance of the material on the impedance of the complete absorber in the following way. The absorber is considered to be a multi-layer system of infinite extent and expressions are obtained for the impedance of such a system with and without a layer of foam. Then by comparing the two expressions it is possible to deduce the effect of the foam.

The acoustic impedance presented to a wave striking the front face of a layer (Z_v) is related to that presented at the back of the same layer (Z_u) by the equation, [7]

$$\frac{Z_v}{W_v} = \frac{Z_u \cosh jk_v l_v + W_v \sinh jk_v l_v}{Z_u \cosh jk_v l_v + W_v \cosh jk_v l_v}, \tag{37}$$

where the suffix v refers to the layer in question, and the suffix u to its backing layer. l_v and k_v are respectively the thickness and propagation constant of the layer. It follows that if a layer is rigidly backed, the impedance of that layer is

$$Z_v = W_v \coth jk_v l_v. \tag{38}$$

By applying equations (37) and (38) to the system illustrated in Figure 1 it can be shown that for $d \ll C/\omega$

$$Z_2 = Z_1 + j\omega M = j\left(\omega M - \frac{\gamma_0 P_0}{\omega d}\right). \tag{39}$$

This is the same equation as would be obtained if the absorber were considered as a simple undamped mass-spring system.

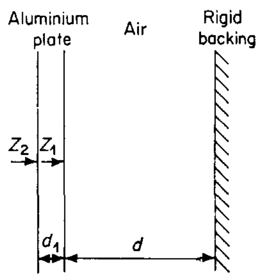

Figure 1. Multilayer representation of the absorber.

Now consider a layer, l_2, of foam to be inserted into the air space as shown in Figure 2. By applying equation (37) to this system, noting that $(\rho_0 C)^2$ is very small compared with Z_G^2 and $\rho_0 C \coth jk_a l_1$ is Z_G, the impedance of the air layer if it is rigidly backed,

$$Z_4 = (j\omega M + Z_G) - \frac{Z_G^2}{Z_G + Z_f}. \tag{40}$$

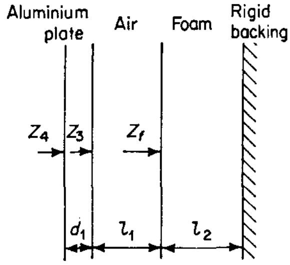

Figure 2. Multilayer representation of the absorber with a foam infill.

Comparing equations (39) and (40), it is seen that $(j\omega M + Z_G)$ is the impedance of an absorber of depth l_1, an accurate value of which, taking into account the stiffness of the panel, can be obtained using equation (34). The term $Z_G^2/(Z_G + Z_f)$ is evaluated from a knowledge of Z_f, which can be obtained by experiment, and the fact that

$$Z_G = -j\rho_0 C \cot k_a l_1. \tag{41}$$

The acoustic impedance of the complete absorber is then obtained by adding these two terms vectorially.

3. EXPERIMENTAL

3.1. METHOD

The construction of the absorber is shown in Figure 3. Its specific acoustic impedance was measured using a standing wave interferometer. This is of normal construction except that the measuring tube is of square cross-section, so that the absorber can be clamped on the end with only its absorbing surface presented to waves travelling along the tube. Because in many instances the absorption to be measured was very small it was necessary to determine the tube absorption α very precisely. Consequently, a rather unusual method was developed in which α was obtained at each frequency by measuring the standing wave ratio with the sample replaced by a rigid block of metal. The standing wave ratio was then measured with the sample present, S_1, and hence knowing α a value was obtained for the reflection coefficient r, from the equation, [8]:

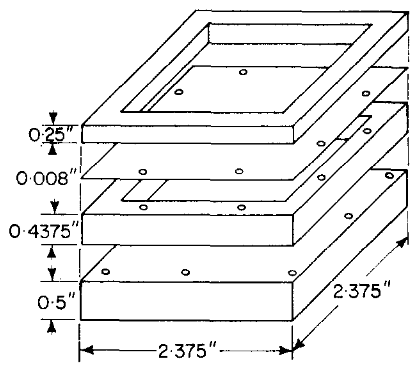

Figure 3. Construction of the panel absorber.

$$S_1 = \left[\frac{e^{2\alpha D_m} + r^2 e^{-2\alpha D_m} + 2r\cos\left(\frac{\alpha}{2kr}(e^{2\alpha D_m} - r^2 e^{-2\alpha D_m})\right)}{e^{2\alpha x_0} + r^2 e^{-2\alpha x_0} - 2r\cos\left(\frac{\alpha}{2kr}(e^{2\alpha x_0} - r^2 e^{-2\alpha x_0})\right)} \right]^{1/2}, \tag{42}$$

where D_m is the distance to the nearest pressure maximum from the sample face and x_0 is the distance to the nearest pressure minimum from the sample face. The method of obtaining r was necessarily an iterative one and was performed on a digital computer. The acoustic impedance of the absorber was then calculated using the usual expressions involving r and the phase change at the sample face. This method proved to be very accurate over the frequency ranges of interest and measurements of large impedances were successfully carried out.

3.2. LOSS FACTOR OF THE PANEL

In order to evaluate equation (34) a value is required for the loss factor of the panel in each mode, g_{mn}. These were found by driving the panel with an electromagnetic transducer, with the back-plate of the absorber removed, and measuring (a) the bandwidth at $1/\sqrt{2}$ of the maximum velocity, and (b) the centre frequency of each resonance. The loss factor was then calculated from the equation, [9]:

$$g = \frac{\delta\omega}{\omega_0}. \tag{43}$$

It was found that this procedure could easily be carried out for the fundamental resonance but the (1, 3) + (3, 1) and (1, 3) − (3, 1) modes were indistinguishable; also the results for the (3, 3) mode were slightly variable. It was therefore assumed that the loss factor in each mode was the same as that in the fundamental, which was found to be 0·005. This value should be entirely due to internal losses in the panel and its support and not to any radiation damping, as the path length between the front and back faces of the panel during the experiment was very small compared with the wavelengths of sound of interest. It would be expected therefore that the loss factor would not be altered by the attachment of the back plate to form the complete absorber. The assumption for the higher order modes, however, was not fully justifiable but was somewhat vindicated later by the good agreement obtained between the calculated and measured values of impedance. Also, when foam is introduced into the absorber the hysteretic damping becomes only a small part of the total damping and does not need to be known to a high degree of accuracy.

3.3. EXPERIMENT A

This experiment was performed to test both the theory of the absorber without any infill, and the reliability of the experimental technique. The acoustic impedance of the absorber was measured over two frequency bands, namely 500 to 1200 Hz and 2200 to 2500 Hz, which lie around the first two calculated natural frequencies. Measurements in between these two bands were unsuccessful as the impedance of the absorber in this frequency range is extremely high. The experiment was repeated four times, the absorber being reassembled each time.

The results of the experiment are shown in Figures 4 and 5 in which they are compared with the calculated values of impedance and absorption coefficient, respectively. From these figures it can be seen that

(i) two resonances were observed, one at approximately 805 Hz and the other at approximately 2240 Hz; these agree closely with the calculated values of 807 and 2251 Hz;
(ii) at frequencies on either side of the resonances the measured values agree closely with the calculated values;
(iii) the measurements are reproducible.

It is of interest that under these conditions the absorption peak at the second resonance is higher than at the fundamental. However, both peaks are very sharp and of little use for absorption purposes. Also, at the fundamental resonance the panel stiffness and cavity stiffness are almost identical and affect the resonant frequency equally; while at the second resonance the panel stiffness is by far the greater of the two and so controls this resonant frequency.

3.4. EXPERIMENT B

In this experiment the impedance of the absorber was measured over the lower frequency range, with three different thicknesses of polyurethane foam against the back wall of the cavity. Then the impedance of the foam alone was measured at the same frequencies, and by using equation (40), theoretical values were obtained for the impedance of the absorber.

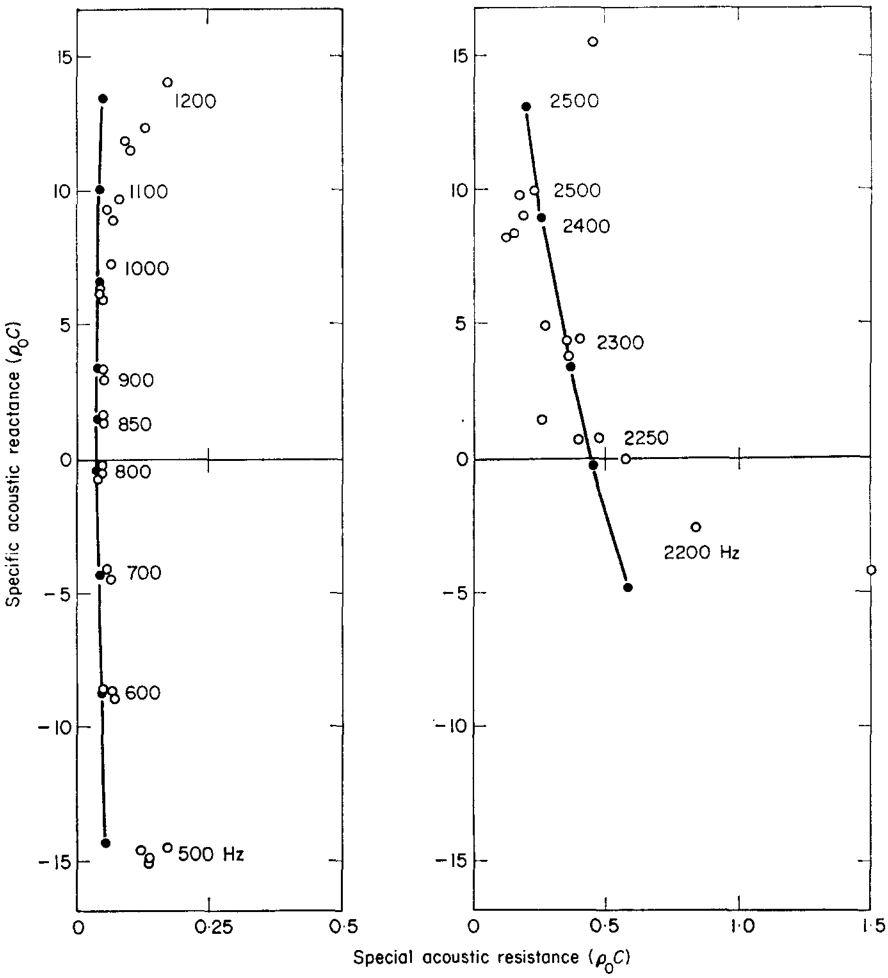

Figure 4. Impedance diagrams of the absolute without foam, showing both the experimental and calculated values. ○, Experimental values; ●, calculated values.

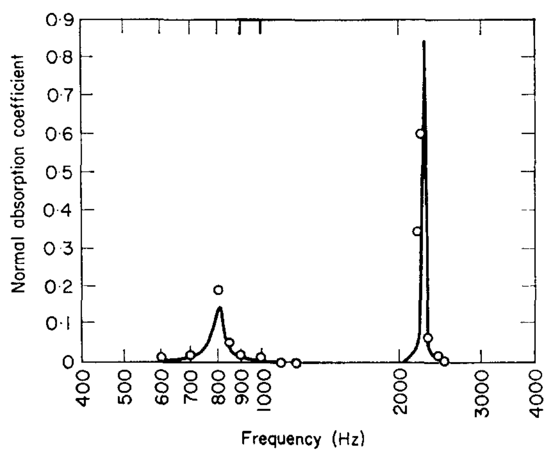

Figure 5. Comparison between measured and calculated values of absorption coefficient for the absorber without foam infill. ○, Experimental values; ——, calculated curve.

The calculated and measured impedances are compared in Figure 6, from which it can be seen that good agreement was obtained between the two sets of values. However, a small

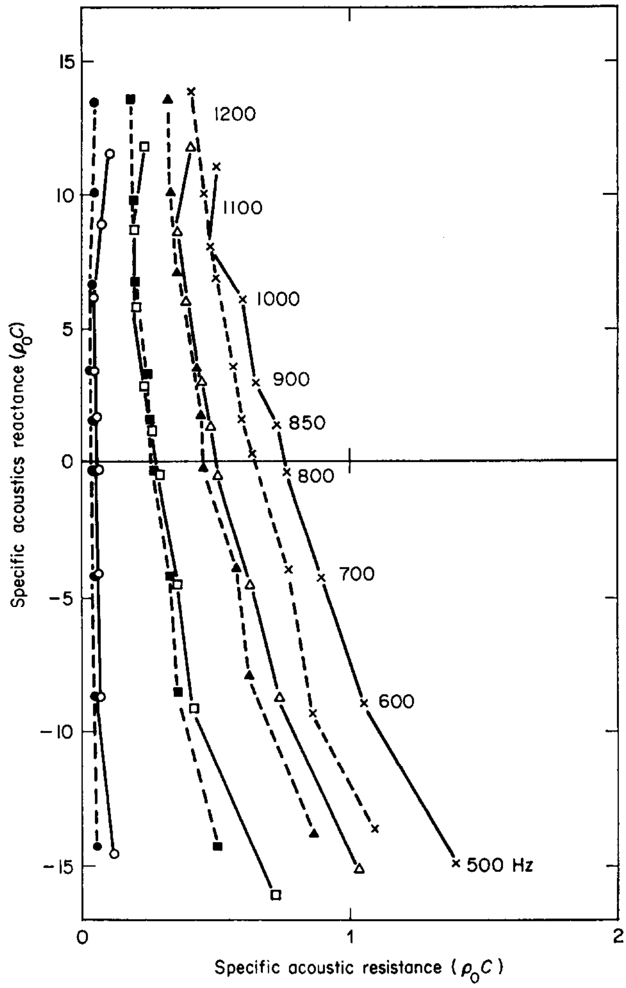

Figure 6. Impedance diagram of the absorber with foam showing both the experimental (—o—o—) and calculated (--●--●--) values.
●, o, Without foam; ■, □, foam = 0·125 in. thick; ▲, △, foam = 0·250 in. thick; X, foam = 0·375 in. thick.

systematic error does appear to be present as the difference between the values is less at high frequencies than at low frequencies, and for shallower infills of foam than deeper ones.

4. CONCLUSIONS

Equations have been derived which describe the variation with frequency of the normal incidence acoustic impedance of a panel absorber. The equations take into account not only the stiffness and loss factor of the panel but also the effect of introducing flexible ployurethane foam into the cavity of the absorber. Experimental values of impedance obtained on a square absorber with an aluminium facing, with and without additional damping, agree very well with the theoretical values.

The advantages of using a stiff panel absorber are that it can have more than one resonant frequency and hence more than one absorption band, and that it is easy to construct offering a wide choice of materials. The disadvantages are that the stiffness of the panel reduces the width of the absorption bands and that the damping can only be optimized for one of the resonant frequencies.

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APPENDIX

NOTATION

a, b	sides of panel
A_{mn}, B_{mn}, c	modal constants
C	velocity of sound in air
d	depth of cavity
d_1	thickness of panel
D	bending stiffness of panel
e	exponential
E	Young's modulus of panel
$f(x)$	mode shape in x-direction
F	driving force
g_{mn}	loss factor of panel
$g(y)$	mode shape in y-direction
j	square root of -1
k	modal constant
k_a	wave number in air
l_1, l_2	thicknesses of air and foam layers
m	mode number
M	mass/unit area of panel
n	mode number
p	sound pressure
P_0	atmospheric pressure
R_a	resistance of absorber
t	time
T_p	kinetic energy of panel
U_a, U_p	potential energies of cavity and panel
v	volume of cavity
V_{mn}, V_s	velocity amplitudes
w_{mn}	panel displacement
W_u, W_v	characteristic impedances
W_{mn}, W_s	displacement amplitudes
x	axis in plane of panel

X_a	reactance of absorber
y	axis in plane of panel
z	axis perpendicular to panel
Z_a, Z_f, Z_G	impedance of absorber, foam, and air layer
γ_0	ratio of specific heats of air
γ_m	modal constant
ϵ_n	modal constant
$\delta\omega$	bandwidth at peak amplitude
η	integer
ρ_0	mean density of air
σ	Poisson's ratio of panel
ϕ	maximum panel displacement
Φ	maximum displacement amplitude
ω	angular frequency
ω_0	natural frequency