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CALCULATING THE ACOUSTICAL ROOM RESPONSE BY THE USE OF A RAY TRACING TECHNIQUE

A. KROKSTAD, S. STRØM AND S. SØRSDAL

Acoustical Laboratory, The Technical University of Norway, Trondheim, Norway

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The distribution of early reflected sound over the audience areas in concert halls is investigated, especially with respect to the shape of halls. The study is based on geometrical acoustics, using a ray tracing technique. The sound intensity is calculated by digital computer, and a graphical representation obtained. Test results for a rectangular and a fan-shaped hall are given.

1. INTRODUCTION

The use of a digital computer in room acoustics opens possibilities for the prediction of the acoustical behaviour of halls. Previously, such methods have been of theoretical interest only, due to the required laborious calculations. Some of the methods were listed by Schroeder, Atal and Bird [1] at the 4th International Congress on Acoustics, 1962. Later publications on these lines are relatively scanty, perhaps due to the fact that the evaluation of results poses great problems. To a certain degree, a simulation technique as described in reference 1 may short circuit this stage. This may be of great practical assistance in testing the ideas of the architect. In the systematic synthesis of suitable hall shapes, it will be necessary to study trends in the variation of objective data, preferably data which may be evaluated by the use of well-established criteria. For numerical calculation we assume that the time and space distribution of early reflected sound, here called room response, will provide valuable data in addition to such purely statistical parameters as reverberation times. We prefer a deterministical approach to the Monte Carlo method.

2. BASIC ACOUSTICAL PRINCIPLES. A BRIEF DESCRIPTION OF THE INVESTIGATION AND A REVIEW OF RELEVANT EARLIER WORK

The basis of our calculations is that of geometrical acoustics as used extensively in optical model experiments in which the space distribution patterns of early reflected sound are investigated.

Using ray tracing, calculation enables us to study, simultaneously, the space and time distribution of the early reflections. The results of such calculations may provide valuable information when one considers the relations established by Haas [2], Lochner and Burger [3] and Seraphim [4]. These connect intensity and relative time delay for speech in auditoriums. Similar evaluations for concert halls are under development. The "initial gap" as introduced by Beranek [5] and the qualitative "Christmas Tree" criterion [6] may be considered as parts of such a criterion.

A more detailed psycho-acoustical investigation was performed by Seraphim [4]. Marshall [7] introduced "spatial responsiveness" as a desirable property of concert halls. Considerable experimental work has also been applied to the measurement of the pulse response of auditoriums and concert halls. Echo diagrams as recorded directly on a CRT were published by Junius [8]. More recently, Preizer [9] gave a detailed account of the smoothing of the spatial fluctuations, evaluating the experimental results in terms of a few numerical parameters.

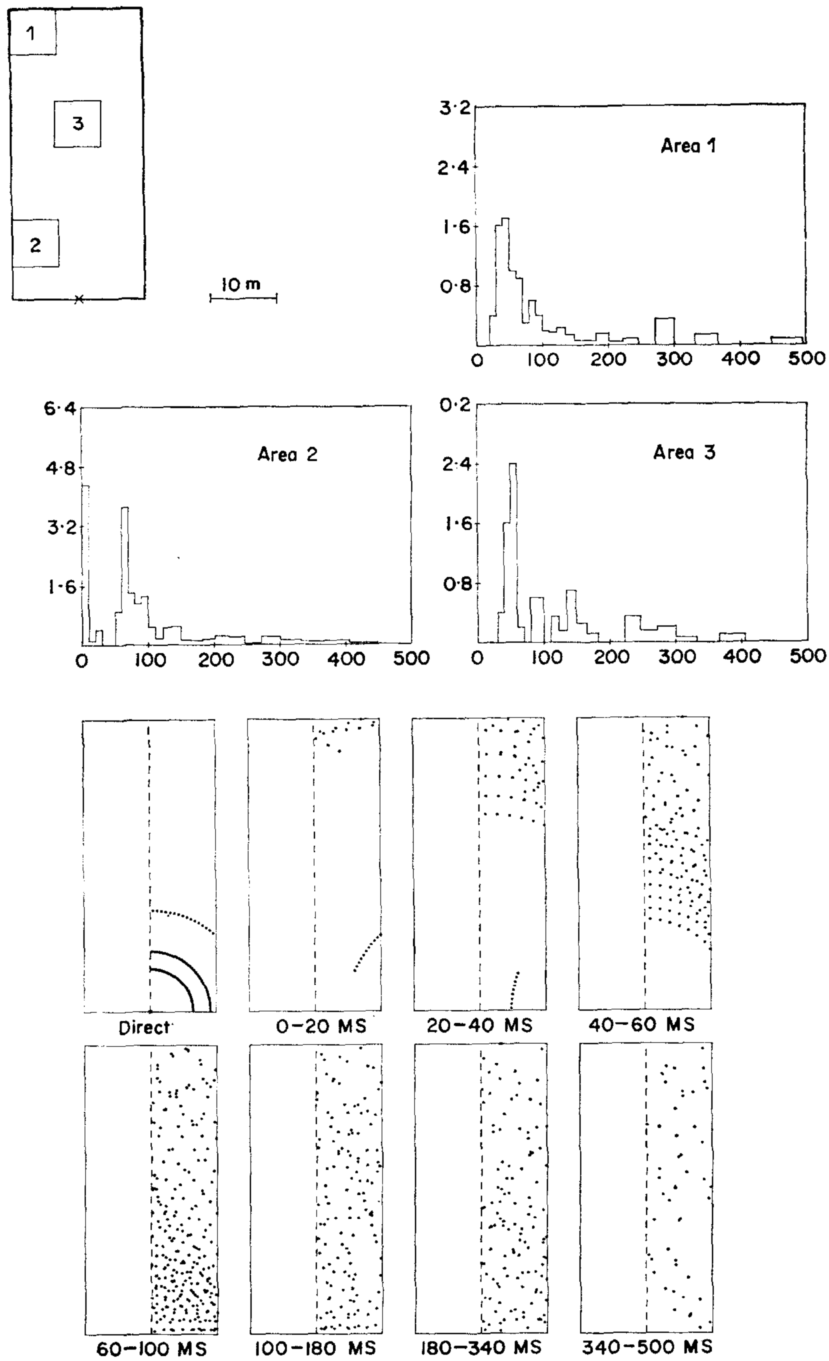

Figure 1. Rectangular hall. Echo diagrams for three audience areas are shown in the upper part of the figure. Unit of abscissa is 1 msec, unit of ordinate is 1 strike/msec. The lower part shows an alternative presentation of the room response. Height of the hall: 16 m. The sound source (×) is located 1 m above the floor. Plotting of direct sound near to the source is suppressed.

The purpose of this restricted study is to investigate the influence of the main dimensions and geometrical shape of a hall upon the pulse response and its variation over the audience area. The calculation procedure uses a mathematical model of a hall, which is excited by a

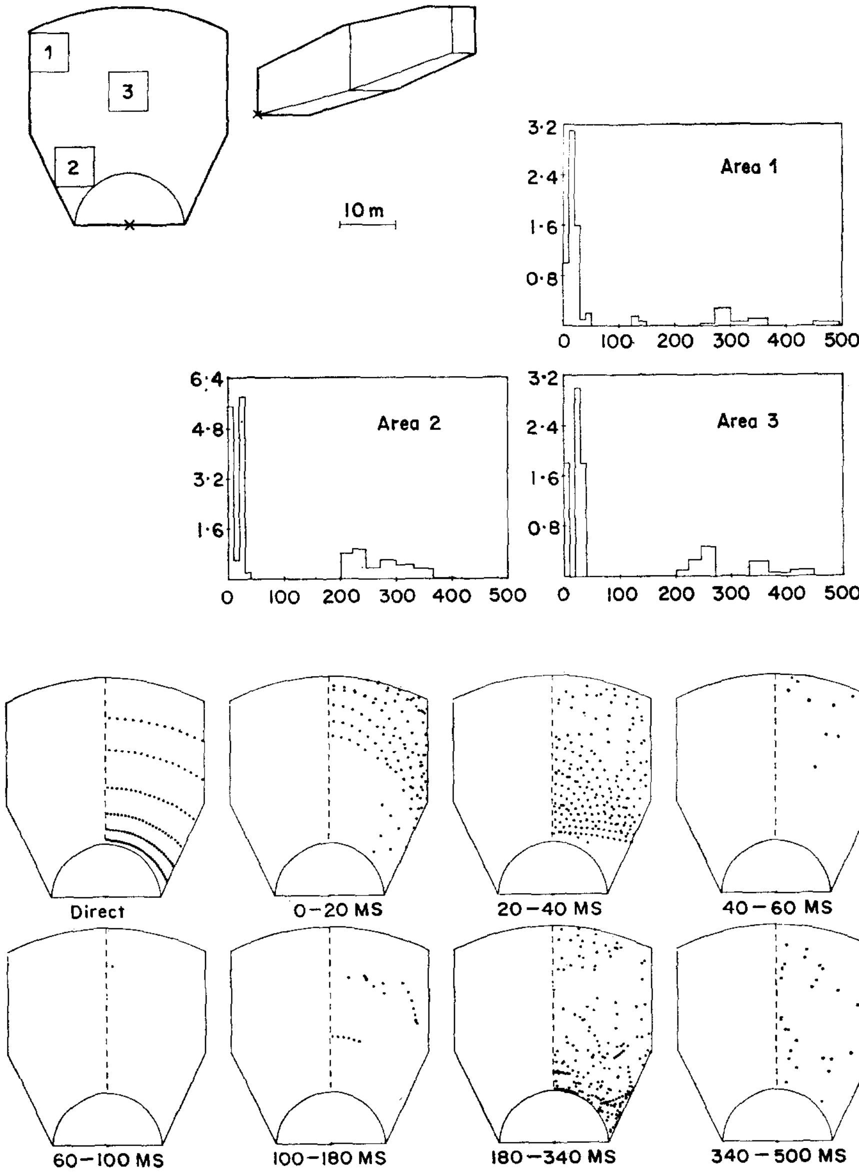

Figure 2. Partly fan-shaped hall. Upper part of the figure shows echo diagrams. Unit of abscissa is 1 msec, unit of ordinate is 1 strike/msec. Lower part shows room response. The sound source (×) is located on the floor.

sound pulse emitted from a fixed point source. Energy is represented by rays equally distributed over the whole or over a selected part of the solid angle. The life history of each ray is calculated assuming geometrical reflection at all surfaces, until the ray strikes the audience area where it is assumed to be totally absorbed. The points of impingement for all emitted rays and the time delay of the impingement relative to direct sound are calculated. The data are stored and processed to give two different kinds of output, both plotted (directly) by a drafting machine connected to the computer. Some other data are also stored for possible later use: e.g., the total number of strikes upon the various reflecting surfaces.

The outputs shown in the upper parts of Figure 1 and Figure 2, resemble conventional echo diagrams or pulse responses. As the rays are assumed to represent equal energy quanta, energy integration in space and time is performed simply by counting strikes. The limited number of rays necessitates the summation over relatively large areas; the time intervals are also increased with time. The areas are indicated in the drawing. Time delay intervals of

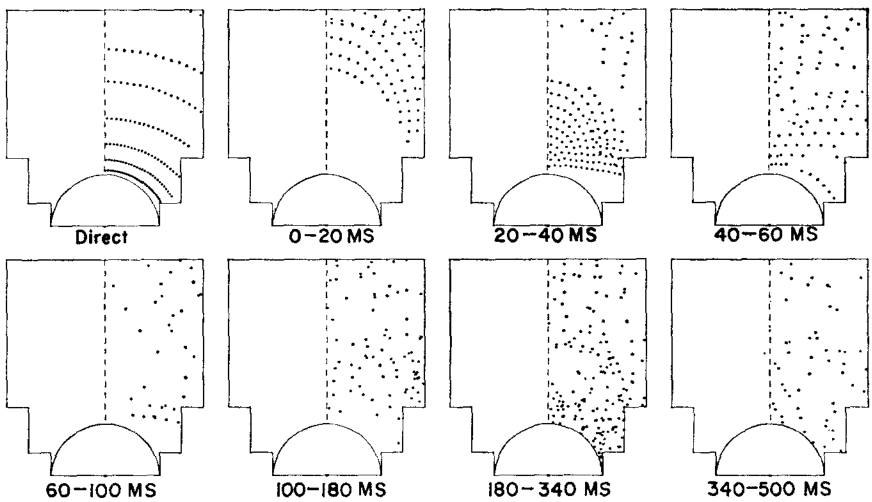

Figure 3. Room response of a partly fan-shaped hall. Floor and ceiling are similar to those of the hall shown in Figure 2. The sound source (×) is located on the floor.

10 msec are used below 100 msec, increasing linearly with time delay above 100 msec. The unit of the ordinate axis is one strike per msec; the unit of the abscissa axis is 1 msec.

When data are presented in echo diagrams, one may introduce the actual absorption coefficients of the various surfaces.

The second kind of output, as shown in the lower parts of Figure 1 and Figure 2 and in Figure 3, is obtained by plotting the striking points in the audience areas in various time-delay intervals. The density of points compared to the length of the time intervals gives an indication of the sound intensity. This integration assumes equal energy per point; thus the actual absorption coefficients of the surfaces cannot be taken into account.

3. DEMONSTRATION OF THE METHOD APPLIED TO TYPICAL SHAPES

The procedure described in section 2 and later in section 4, has been programmed for a digital computer (UNIVAC 1107). A drafting machine (KINGMATIC 1215) is used to present the results, as shown in Figures 1–3.

Figure 1 is data from a rectangular hall of main dimensions typical of halls of the 19th century. Symmetry is assumed, the sound source being located on the back-stage wall one meter above floor level. The space-time distribution of rays striking the audience area is shown in the lower part of Figure 1. The time delay relative to the direct sound is divided in eight groups with a range of 20 msec for small delays, increasing for greater delays. The density of dots is a linear measure of acoustic intensity. Due to the position of the source near to the floor, the intensity of direct and early reflected sound is small. After a delay of 40 msec the diagrams show a relatively even space distribution of impinging energy and an intensity which decreases rather evenly with increasing time delay. Details of the time distribution of energy are shown more clearly in the upper part of Figure 1, in which an integration is performed over square areas of the audience, giving impinging energy as a function of time delay. This type of diagram may be compared to experimental echo diagrams. The linear intensity scale of the ordinate is comparable to linear sound pressure scales in experimental recordings. Evaluated by a "Christmas Tree" criterion, the distribution of this hall shape seems quite reasonable, but an echo from the back wall may be perceived in area 2.

The "old-fashioned" hall may be directly compared to a design preferred in our century as shown in Figure 2. The hall is partly fan shaped, has a cylindrical back wall and a width which is great compared to the height. Two inclinations of the audience areas secure an even distribution of direct sound. The great problem of this shape is a lack of energy in the time delay interval 40–180 msec (Figure 2, lower part). The energy in the interval 180–340 msec is unevenly distributed, partly due to the cylindrical back wall, but mostly due to the fan shape. Even with a reverberation time in the range of 2 sec, this hall will sound "dry" due to the great intensity of direct and early reflected sound as compared to multi-reflected sound.

A great improvement may be obtained by breaking up the oblique walls in rectangular steps as shown in Figure 3, resulting in a far better distribution of energy in space and time. It is rather important to note that these details may change the energy balance between early reflected and diffuse sound. A more detailed investigation into these problems will be presented in a later publication.

4. DETAILS OF THE COMPUTATION PROCEDURE

To obtain a reasonable degree of precision, it is necessary to avoid too small surfaces in the room. We therefore have to simplify the architectural drawings. The surfaces of the rooms described in section 3, are parts of planes, cones and cylinders.

The direction of the rays is computed from a set of equations given in a recent paper by Stenseng [10]. His intention was to derive a uniform distribution of a limited number of points on a unit sphere. The rays all start from the centre of the sphere and pass through these points. The computations are performed in polar co-ordinates, only the first octant being used (cf. Figure 4). Between $z = 0$ and $z = 1$ we introduce n planes parallel to the xy plane. The angle θ_i in Figure 4 is given by

$$\theta_i = \frac{2i-1}{2} \cdot \frac{90^\circ}{n}, \qquad i = 1, 2, \ldots, n. \tag{1}$$

The circles have the radius $r_i = \sin\theta_i$, and their arc in the first octant is $l_i = \frac{1}{4} . 2\pi . r_i . \sin\theta_i$. The distance on the sphere between two adjacent circles is $b = \frac{1}{2}\pi/n$. We stipulate m_i points on the ith circle, where m_i is taken to be the integer approximating l_i/b most closely. The angle $\phi_{i,j}$ in Figure 4, is now given by

$$\phi_{i,j} = \frac{2j-1}{2} \cdot \frac{90^\circ}{m_i}, \qquad j = 1, 2, \ldots, m_i. \tag{2}$$

For every i and j, the corresponding ray has the direction cosines

$$\left.\begin{aligned} \alpha_{i,j} &= \sin\theta_i . \cos\phi_{i,j}, \\ \beta_{i,j} &= \sin\theta_i . \sin\phi_{i,j}, \\ \gamma_{i,j} &= \cos\theta_i. \end{aligned}\right\} \tag{3}$$

Because we approximate l_i/b to an integer m_i, we obtain a non-uniform distribution of points, but the distribution may be expected to be fairly good if we use a considerable number of points.

The rays have the equation

$$\frac{x-x_0}{\alpha}=\frac{y-y_0}{\beta}=\frac{z-z_0}{\gamma}. \tag{4}$$

Here (x_0, y_0, z_0) are the co-ordinates for the starting position of the ray, while (α, β, γ) determine the direction. The points of reflection (or absorption) are found by combining equation (4) with the equation for a surface. The impact is a real one if it lies on that part of the surface which forms the boundary of the room. If more than one possible point is found, the point lying nearest to (x_0, y_0, z_0) is chosen.

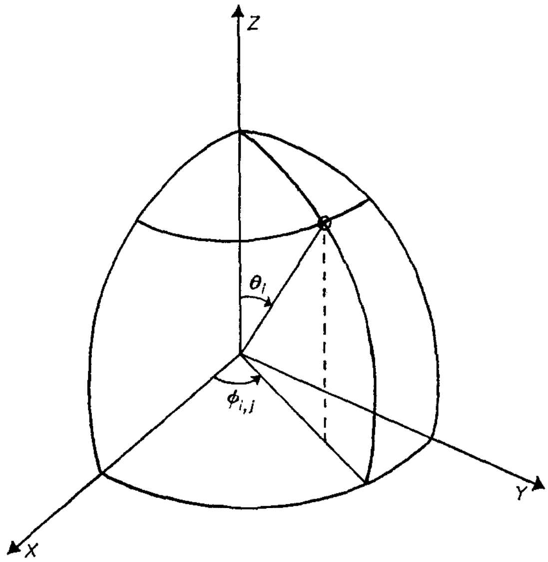

Figure 4. Co-ordinate systems.

When, for instance, a ray is reflected by a plane, the new direction cosines are

$$\left.\begin{aligned}\alpha_2&=\alpha_1-2.\cos u.\mu,\\ \beta_2&=\beta_1-2.\cos u.\eta,\\ \gamma_2&=\gamma_1-2.\cos u.\xi.\end{aligned}\right\} \tag{5}$$

Here $(\alpha_1, \beta_1, \gamma_1)$ are the old direction cosines, u is the angle between the ray and the normal to the plane, and (μ, η, ξ) are the direction cosines of the normal. If the ray is reflected by a cylindrical wall, we first have to compute the equation for the tangential plane at the impact point, and then continue with equation (5).

A given ray is repeatedly reflected until it is absorbed or becomes longer than a given maximum. The computer stores the data for the impact upon the audience area and the number of reflections at various surfaces.

5. DISCUSSION OF METHODS AND RESULTS

Because the method is based upon geometrical acoustics it suffers from the limitations inherent in this approach. Perhaps the most severe restriction is the limited frequency range over which the results are valid. It is an underlying assumption in all methods using sound rays that the wavelength corresponding to the lowest frequency of the sound be small compared to the linear dimensions of the room and its surfaces. When the room is fairly large and its shape is simplified, as in the halls described in this article, the results may be valid for frequencies above 100 Hz. However, in the actual design of a hall, one may also wish to know the effects of changes in details, such as the introduction of diffusing wall elements. Even if such details can be included in the mathematical model of a hall without causing excessive computing time, the validity range of the results may be difficult to state. In this connection it might be noted that the low frequency content of early reflected sound seems to have little influence upon the perception of sound in live rooms, provided that there is "free sight" to the sound source [11].

Uncertainties are also introduced in the results by the use of a limited number of discrete rays to represent the sound. As the angle between adjacent rays remains roughly constant, and since all rays are radiated from a point source, this representation gradually becomes less exact with increasing ray length.

As an example, the arc between adjacent rays at a radius corresponding to 500 msec travel time will be approximately one-fifth of the overall length of the halls shown in this article. Even with the simplifications employed in describing the hall, this uncertainty is quite large. It is probable, however, that such a decreasing accuracy may not invalidate the results. At large travel times when the sound field tends to be diffuse, the results can only be given as statistical averages. The uncertainty can be reduced at will, but only at the expense of increased computing time.

At the present state of program development some uncertainty is also introduced by the method by which the direction of the sound rays is determined (section 4). The degree of unevenness in the distribution of directions has not yet been thoroughly investigated. Improvements are no doubt possible; the main problem is to describe in suitable mathematical form what is meant by "even distribution" in this case. A Monte Carlo method for computing an even distribution of directions in the sense of "equally probable" may easily be found, but the authors consider the deterministic approach to be much more efficient.

As mentioned in section 2, the surfaces of the room are considered to be either totally reflective or totally absorbent. Other absorption characteristics of the surfaces, together with air damping, may readily be included in the computations. The authors believe that these factors are of minor interest in view of the building materials commonly used in concert halls.

In spite of the limitations and uncertainties of the methods described, it seems obvious that they may provide useful information concerning the transient behaviour of sound in rooms. The full use of such computations can be made when they are followed by a digital simulation of actual sounds, including the reverberation process, as described by Schroeder *et al.* [1]. It may then become possible to perform subjective evaluations of rooms with their shape, surface materials and the listening position as possible parameters.

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