J. Sound Vib. (1967) **6** (3), 324–334

THE TRANSMISSION LOSS OF DOUBLE PANELS

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(*Received* 16 *February* 1967)

Existing theories relating to the transmission loss of double panels are studied. It is shown that it is not possible to obtain agreement between theoretical and measured curves for random incidence transmission loss without the use of terms other than the mass of the panels and their separation. These extra terms are often of doubtful origin. A new theory is discussed which, in its simplest form, agrees with the prediction of the other theories. With this new theory it is possible to introduce as an extra term the addition of sound-absorbing material in the cavity between two impervious septa. Using this extra factor, the absorption coefficient of the cavity walls, it is possible to obtain good agreement between theoretical predictions and observed values of transmission loss. When applied to cavity brick walls the theory predicts values of transmission loss far in excess of most of the measured values. This implies that the insulation of a brick wall is determined solely by flanking paths and vibration bridging and not by the inherent insulating properties of the wall as is the case with simple "mass law" panels.

1. INTRODUCTION

Two models (Beranek and Work [1] and London [2]) have been used to derive theoretical expressions for the transmission loss of double panels (panels having two independent septa separated by an air gap). This paper considers these models and shows that for the simple case of mass law septa and an air gap the models predict identical curves for transmission loss. A third model is described which, in simple form, gives the same predictions for transmission loss as the first two models. However, this third model permits exploration of the effect of adding on of an absorbing surface in the cavity; a promising correlation with experimental results is then found.

2. THE GENERAL APPROACH TO THE PREDICTIONS OF DIFFUSE FIELD TRANSMISSION LOSSES

The theoretical transmission loss of a panel for random incidence sound can be deduced in two ways. These are the mode/coupling method and the diffuse field theory. The mode/coupling method (for a recent discussion see White and Powell [3]) derives the transmission loss of a panel by considering the degree of coupling between room modes in the transmission and reception rooms and the modes of vibration of the intervening panel. Considerable difficulties arise with this method: a very large number of individual modes are excited in a given one-third octave band and there are doubts about the possibility of exciting high-order modes with incoherent white noise sources. The authors decided to use the second method exclusively.

The diffuse-field method first requires the derivation of the transmission coefficient of the panel as a function of frequency and angle of incidence, $\tau(\theta, \omega)$.

$\tau(\theta, \omega)$ is then integrated over a range of angles to give the mean transmission coefficient (see Appendix for a list of symbols used),

$$\bar{\tau}(\omega) = \frac{\int_0^{\theta_l} \tau(\theta, \omega) \cos\theta \sin\theta \, d\theta}{\int_0^{\theta_l} \cos\theta \sin\theta \, d\theta}. \tag{1}$$

θ_l is called the limiting angle above which it is assumed no sound is received, and varies between 70° and 85°. The easiest way to show that this treatment is reasonable is to plot the angles of incidence of all the modes in a one-third octave band (see Figure 1). The figure shows that the number of modes in a small band increases gradually as angle increases and then falls sharply, there being no sound incident above about 84°. In a room containing absorption the higher-order modes with high angles of incidence are more heavily damped and are of lower intensity than the lower modes; the value of θ_l decreases with the reverberation time. Equation (1) is thus seen to be approximately correct. The random incidence transmission loss of a panel is then found from the equation

$$TL(\omega) = 10 \log[1/\bar{\tau}(\omega)]. \tag{2}$$

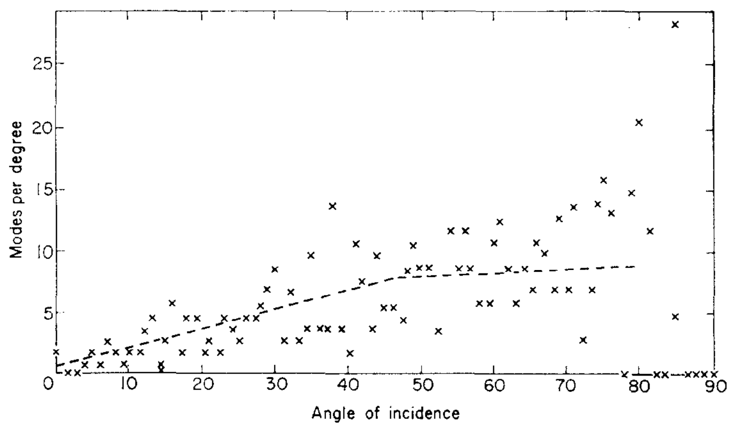

Figure 1. Variation of mode density with angle of incidence.

3. BERANEK AND WORK'S THEORY

Beranek and Work's theory [1] has the advantage that it is theoretically exact. Solutions of the wave equation are stated for the various regions of a multiple panel and the arbitrary constants involved in these solutions are evaluated by means of the principle of the continuity of acoustic impedance at the interfaces of the various media. Because of this exactness of Beranek and Work's solution we must use results predicted by this theory as a guide to evaluating the worth of alternative formulae. Beranek and Work give equations that can be used to cover a large number of possible panel constructions for normally incident waves. When appropriate values are put into their equations an expression for the sound pressure ratio across a double panel of surface mass M and air gap width d is found:

$$\frac{p_t}{p_{\text{face}}} = \frac{\rho c \coth(jkd + \Phi)}{(\rho c \coth(jkd + \Phi) + j\omega M)} \cdot \frac{\cosh \Phi}{\cosh(jkd + \Phi)} \cdot \frac{\rho c}{(\rho c + j\omega M)}, \tag{3}$$

where

$$\Phi = \operatorname{arcoth}(1 + j\omega M/\rho c). \tag{4}$$

Here p_t is the transmitted wave pressure and p_{face} is the pressure on the incident face of the panel. Beranek and Work's theory allows us to calculate the face impedance of the panel to incident waves, Z. This is

$$Z = j\omega M + \rho c \coth(jkd + \Phi), \tag{5}$$

and since the ratio of the incident sound pressure, p_i, to the pressure on the face of the panel of impedance Z is

$$\frac{p_{\text{face}}}{p_i} = \frac{2Z}{(Z+\rho c)}, \tag{6}$$

the transmission factor for a double panel will be

$$\tau = \left|\frac{p_t}{p_i}\right|^2 = \left|\frac{2\rho c \coth(jkd+\Phi)}{[\rho c \coth(jkd+\Phi)+j\omega M+\rho c]}\cdot\frac{\cosh\Phi}{\cosh(jkd+\Phi)}\cdot\frac{\rho c}{(\rho c+j\omega M)}\right|^2. \tag{7}$$

A curve derived for the normal incidence transmission loss of a double panel of septa surface mass $M = 0{\cdot}5$ g/cm^2 and separation 30·5 cm is shown in Figure 2, curve (1). Curve (1) shows two zero points, one at about 68 Hz and the other at 575 Hz. The lower zero occurs at the resonant frequency of a pair of masses equal in weight to a unit area of the panel connected by a spring of stiffness equal to the "stiffness" of the air between the panels. This frequency, the "lower London frequency", as we shall call it, is given by the equation

$$LLf = \frac{1}{2\pi}\sqrt{\left(\frac{2\gamma p}{Md}\right)}. \tag{8}$$

It was shown by further analysis that the variation in LLf predicted by Beranek and Work when M and d are varied is in accordance with equation (8). There can be no doubt that the physical mechanism causing this zero is the mass-spring-mass resonance. The significance of this resonance will be discussed later. The zero at 575 Hz occurs just above the first standing wave frequency, 566·4 Hz, where the wavelength is exactly equal to twice the panel separation. Similar dips occur near the integer multiples of this frequency. Beranek and Work initially stated that the resonance peaks occur at the standing wave frequencies and at these frequencies the transmission loss is approximately equal to the corresponding normal incidence mass law value. Our theoretical investigation shows that at the standing wave frequencies the attenuation is indeed equal to the attenuation predicted by mass law. But the minima occur at frequencies slightly greater than the standing wave frequencies, the transmission loss being zero at these points. Beranek corrected the earlier work in the book *Noise Reduction* [4]. Beranek and Work did not develop their theoretical work to include the effect of random incidence fields in the way the present authors have described and we will leave discussion of this aspect until we have reviewed London's theory.

4. LONDON'S THEORY

London [2] considers the problem as a continuity problem with forward and reverse travelling waves between the two septa and a forward wave beyond the second septum. London considers the continuity of wave potential across the septa, treating the individual panels as having an impedance, Z, given by

$$Z = 2R/\cos\theta + j\omega M(1 - f^2 \sin^4\theta / f_c^2). \tag{9}$$

The last term in this definition is introduced to take account of possible coincidence effects due to bending waves in the septa. The first term is London's arbitrary resistance term. He introduced this term to align the theoretical values of transmission loss with the measured values. There appears to be no other reason for the introduction of the R term nor any physical process that would necessitate its introduction.

The expression London derives for the transmission coefficient is

$$\tau = 1/\{1+2y+y^2[1-\exp(-2jkd\cos\theta)]\}, \tag{10}$$

where

$$y = Z\cos\theta/2\rho c. \tag{11}$$

If this expression is plotted out as a function of $\omega\cos\theta$ for given values of M and d, the result obtained is identical to that obtained using Beranek and Work's theory. A curve for $M=0.5$ g/cm^2 and $d=30.5$ cm is shown in Figure 2. All the comments with regard to standing wave frequencies and mass spring resonances made about Beranek and Work's theory are immediately applicable to London's result.

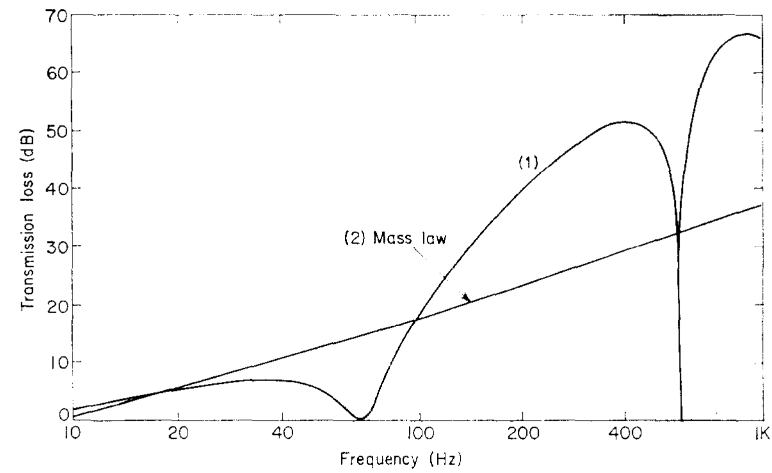

Figure 2. Normal incidence double panel transmission loss $m=0.5$ g/cm^2, $L=1$ ft.

London integrates this expression in the manner indicated in Section 2 of this paper. If we set $R=0$, $f_c=\infty$ and $\theta_l=90°$, curve (2) in Figure 3 is obtained. This curve lies below mass law for all frequencies. London explains this low value of transmission loss in terms of the oblique mass spring resonance. This occurs at a frequency, f, given by

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{2\gamma p}{Md\cos^2\theta}\right)}, \tag{12}$$

for waves incident at an angle θ to the panel. It is seen that in a diffuse field there will always be some angle, θ, for which the transmission loss is zero. London states that low transmission loss values in diffuse field conditions will always occur for $R=0$ because of this effect. By varying the value of R London obtained good agreement between theory and experiment. It is, however, possible to obtain values of transmission loss greater than mass law with $R=0$ by introducing the limiting angle described above. There is then an upper frequency, which we shall call the "upper London frequency" (ULf), above which the oblique mass spring resonance will not occur:

$$ULf = \frac{1}{2\pi\cos\theta_l}\sqrt{\left(\frac{2\gamma p}{Md}\right)}. \tag{13}$$

If the diffuse field integration is confined to the region $0^\circ < \theta < 80^\circ$ a transmission loss curve similar to curve (3) in Figure 3 is obtained. It is seen that above *ULf* the curve rises sharply and soon exceeds the mass law. At high frequencies the curve becomes scattered because of the very sharp dips in the normal incidence curve at just above the standing wave frequencies. This is reflected in the uncertainty in the random incidence curve above 1 kHz.

We are now in a position to compare the London diffuse field result with experimental results. Because of the identical results obtained by using the theories of Beranek and Work and of London (assuming $R = 0$) we can quote a single curve for both these theories.

5. THE EXPERIMENTAL RESULTS AND THE PREDICTION OF THEORY

Measurements were made of the transmission loss of a double aluminium panel. Each sheet of $\frac{1}{16}$ in. aluminium was mounted in an individual frame, one set in the wall of the transmission room and the other in the reception room wall. Since the two rooms are isolated from each other there was a minimum of vibration flanking at the edges of the

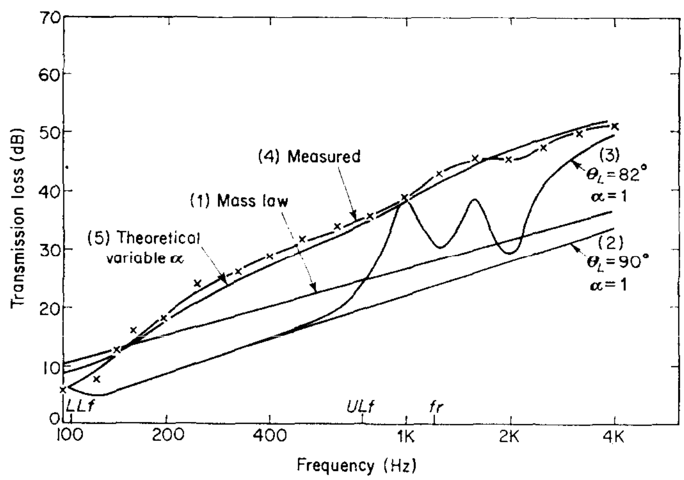

Figure 3. Transmission loss of double aluminium panel.

panels. Measurements were carried out to B.S. 2750 modified according to Mulholland and Parbrook [5]. The separation of the panel was varied over a wide range between 7·1 cm and 39·6 cm. A typical result is curve (4) in Figure 3, which shows that the observed value of transmission loss rises above the random incidence mass law by about 15 dB. It is seen from Figure 3 that the agreement between the measured value of transmission loss and theoretical curves so far considered, curves (2) and (3) for $\theta_l = 90^\circ$ or $\theta_l = 82^\circ$, is poor. Beranek and Work's theory, when integrated, is far below the measured value between *LLf* and *ULf* and also at high frequencies. Agreement might be improved by reducing θ_l but this would not agree with observed and theoretical ideas about θ_l. London obtained a better fit by means of his term but there appears to be no physical mechanism that could be related to R.

In the face of this lack of agreement between existing theory and measured transmission loss a third model for double panel transmission loss was investigated.

6. THE MULTIPLE REFLECTION THEORY

In this theory the sound incident on the panel is treated as a ray which passes through the first panel, being reduced in intensity by a fraction x. x is determined by the mass law theory and is given by

$$x = 1/(1 + j\omega M \cos\theta/2\rho c). \tag{14}$$

A fraction $(1-x)$ of the incident sound is reflected from the first panel at this point. The ray then proceeds across the air space between the two panels and meets the second panel, where further transmission and reflection take place. Successive multiple reflections

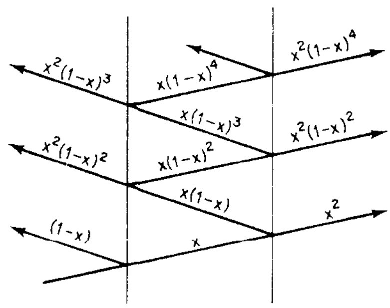

Figure 4. Multiple reflection theory.

occur within the panel such as are shown in Figure 4. Thus the first emerging ray has an amplitude $x^2\phi_I$ (if ϕ_I is the amplitude of the incident beam). The second emerging ray has an amplitude $x^2(1-x)^2$ and is altered in phase by a phase factor, $\exp(-2jkd\cos\theta)$. The third ray has a complex amplitude $x^2(1-x)^4\exp(-4jkd\cos\theta)$. When the sum of the amplitudes of all the emerging rays is taken we find

$$\tau = \left|\frac{\phi_t}{\phi_i}\right|^2 = \left|\frac{x^2}{1-(1-x)^2\exp(-2jkd\cos\theta)}\right|^2. \tag{15}$$

This is the expression for transmission factor which we can integrate to find mean transmission loss. It is easy to show that London's expression,

$$\tau = \left|\frac{1}{1+2y+y^2[-\exp(-2jkd\cos\theta)]}\right|^2 \tag{16}$$

with

$$y = j\omega M\cos\theta/2\rho c, \tag{17}$$

reduces to the multiple reflection expression when the relation $y = (1/x - 1)$ is substituted into London's equation. We thus see that the curve of equation (15) when plotted out will be identical with the curves of London and of Beranek and Work (see Figure 2). The mass-spring-mass resonance is exhibited and the standing waves predicted in the manner already described. This third model, as described so far, then does not predict experimental results any better than the previous models. But the third model does permit examination of the effects of absorbing surfaces in the cavity between the septa. We hoped that this additional physical factor would improve the correlation with experimental work.

7. TRANSMISSION LOSS WITH SOUND-ABSORBING MATERIALS IN THE CAVITY: NORMAL INCIDENCE

If we add sound-absorbing treatments to the inside surfaces of a double wall we can assume as a first approximation that the fraction of sound transmitted by the panel, x, is unaltered but that the fraction reflected, $(1-x)$, is reduced to $\alpha(1-x)$, where α is the reflection coefficient of the sound absorbing material. The series of transmitted waves then becomes:

$$\phi_T = \phi_I x^2[1+\alpha^2(1-x)^2 \exp(-2jkd\cos\theta)+\cdots]. \tag{18}$$

When summed this gives:

$$\tau(\theta) = |x^2/[1-\alpha^2(1-x)^2 \exp(-2jkd\cos\theta)]|^2. \tag{19}$$

Figure 5 shows the variation of normal incidence transmission loss of panels with mass 0·5 g/cm^2 and separation 30·5 cm as a function of frequency and α. There are three

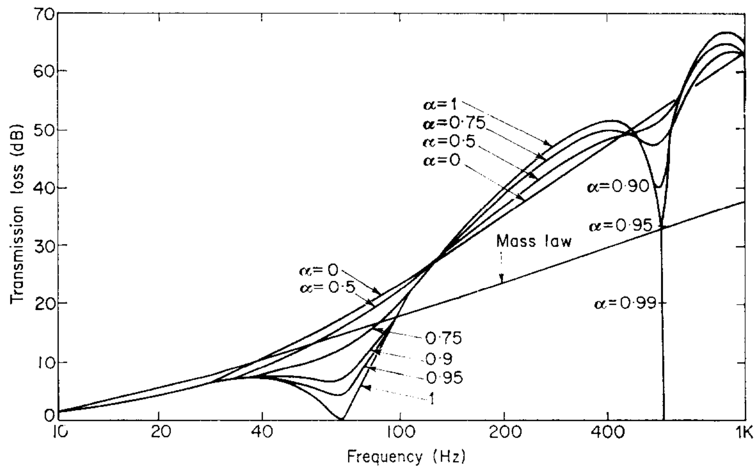

Figure 5. Normal incidence transmission loss, absorbant in cavity.

regions of interest. It is seen that the mass spring resonance dip is filled in gradually, a value of α of less than 0·5 being necessary before the double mass law limit ($\alpha=0$) is approached. At higher frequencies decreasing values of α result in smaller values of transmission loss. This may be explained in the following way: high values obtained when $\alpha=1$ depend on destructive interference between successive components; when $\alpha=0$ and only the first wave passes through the panel there is no destructive interference. In the narrow standing wave resonance band it is seen that even a reflection coefficient of 0·99 is enough to raise the bottom of the dip from zero decibels to twenty decibels and with a reflection coefficient of 0·75 the average insulation is seen to be little different from that with $\alpha=0$. Behaviour for higher standing waves will be similar to that at the first standing wave frequency.

8. TRANSMISSION LOSS WITH SOUND-ABSORBING MATERIALS IN THE CAVITY: DIFFUSE FIELD

If the wall described above ($M=0·5$ g/cm^2, $d=12$ in.) is now placed in a diffuse field with $\theta_l=80°$, the transmission loss curves will be as shown in Figure 6. The lower London frequency is 67 Hz and the upper London frequency is 385 Hz. The first standing wave frequency is 556 Hz. Below the lower London frequency the effect of adding absorption is negligible, but between the two London frequencies the transmission loss is dependent

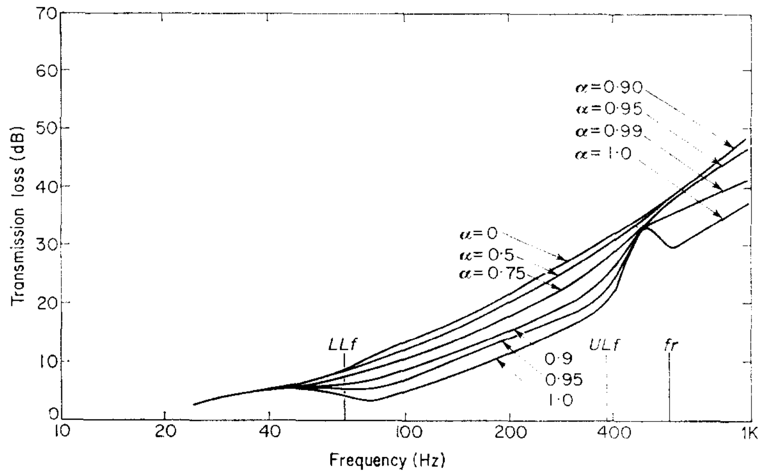

Figure 6. Random incidence transmission loss, absorbant in cavity.

on α. It is seen that only when α is decreased to 0·25 does the value of transmission loss approach the limiting (α=0) value. Above the upper London frequencies the transmission loss curves come together, the lower values rising rapidly towards the limiting value. When the lowest standing wave frequency is reached the curve for α=1·0 falls away and then runs parallel to the limiting curve some 15 dB below it. The curve for α=0·99 also lies below the limiting curve but curves for all other values of α are virtually indistinguishable from the limiting curve.

9. COMPARISON WITH MEASURED TRANSMISSION LOSS

We can now apply the results of this theory to the measured values of transmission loss reported earlier (see Figure 3). By choosing suitable values of reflection coefficient a good fit to the experimental results of curve (4) is obtained. The values of α chosen are shown in Table 1. London can also obtain a fit by use of his *R* term but the α term more nearly

TABLE 1

f	α	*f*	α
100	1	800	0·75
125	0·9	1000	0·85
160	0·7	1250	0·85
200	0·7	1600	0·99
250	0·65	2000	0·99
315	0·65	2500	0·99
400	0·65	3150	0·995
500	0·7	4000	0·995
630	0·7		

corresponds to a physical process occurring in the air gap. The authors consider, therefore, that the α term is to be preferred. This result shows that the theory does give answers that can be made to fit measured values. Further work is needed to confirm that the values of α chosen relate to the sound absorption actually present in the cavity and to study the application of this theory to other isolated double panels. This has been done tentatively by the authors with no outstanding failure as yet.

10. PRACTICAL IMPLICATIONS OF THEORETICAL WORK

The theory indicates that there are three important frequencies associated with a double panel; these are

the lower London frequency, given by $LLf = \frac{1}{2\pi}\sqrt{\left(\frac{2\rho c^2}{Md}\right)}$,

the upper London frequency, given by $ULf = \frac{1}{2\pi \cos\theta_l}\sqrt{\left(\frac{2\rho c^2}{Md}\right)}$,

and the first standing wave frequency, given by $f_R = c/2d$. (20)

Between LLf and ULf the transmission loss is sensitive to sound absorption placed between the panels. In general, the more sound-absorbing treatment that is placed in the cavity the higher will be the insulation of the panel. This is true until the effective reflection coefficient of the panels is 0·25 ,when a limit is reached. Above f_R a completely non-dissipative air gap shows a drop in transmission loss. But only a small amount of sound absorbing treatment (corresponding to $\alpha = 0{\cdot}95$) will raise the transmission loss considerably. After that, however, further treatment has no effect. It is probable that practical panels have enough absorbing properties themselves to have reached the limiting insulation and so the addition of further treatment to panels in this region will be useless. Outside the regions mentioned the addition of sound-absorbing treatment has little effect on transmission loss.

11. TENTATIVE APPLICATION TO BRICK WALL

Figure 7 shows the results of this theory applied to a practical brick wall constructed of two septa of surface mass 24 g/cm^2 with a 2 in. airgap between. For such a system the special frequencies are

LLf	25 Hz,
ULf	144 Hz,
f_R	3400 Hz.

It can be seen from the graphs that the predicted transmission loss is much higher than the values obtained in practice. Curve (1) shows the transmission loss of a double wall measured under normal conditions. It is seen that this curve lies well below theoretical curves (2) to (5). There is, however, a curve reported by Beranek [4] from a paper by Moeller [6] showing the sound insulation between two specially constructed broadcasting studios. In this case the air gap was 12 in. but it is seen that the curve is well above the values obtained in practice [curve (1)], and is in the range of the predicted values for a 2 in. air gap. The implications of these curves are the obvious ones that a 9-in. cavity wall could have a very much higher value of sound insulation than that obtained in practice, and that the insulation of a practical wall is determined by other factors such as flanking, air gaps, ties and mortar bridging rather than by an inherent inability of the panel to provide insulation (such as is the case with a mass law simple panel).

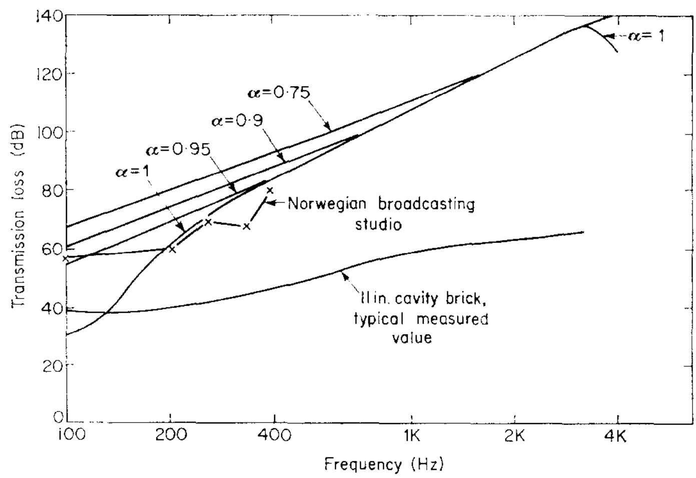

Figure 7. Transmission loss of double brick wall.

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APPENDIX—LIST OF SYMBOLS USED (ORDER OF APPEARANCE)

τ	transmission coefficient
ω	angular frequency
θ	angle of incidence
θ_l	limiting angle of incidence
TL	transmission loss
ρ	density of air
c	velocity of sound in air
p	sound pressure
k	wave number
d	separation of panels
Φ	phase factor
j	$\sqrt{-1}$
Z	impedance
γ	ratio of principal specific heats
LLf	Mass spring mass resonance frequency at normal incidence
ULf	Mass spring mass resonance frequency at limiting angle of incidence

R	London's resistance factor
f	frequency
f_c	coincidence critical frequency
y	impedance ratio
f_R	Lowest standing wave frequency
x	transmission coefficient
α	reflection coefficient