Journal of Sound and Vibration (1990) **141**(2), 323–333

THEORY OF A TWO SOURCE-LOCATION METHOD FOR DIRECT EXPERIMENTAL EVALUATION OF THE FOUR-POLE PARAMETERS OF AN AEROACOUSTIC ELEMENT

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(*Received* 29 *June* 1989, *and in revised form* 16 *November* 1989)

A new method, called the two source-location method, is described for measurement of the four-pole or the transfer matrix parameters for an acoustic element or a subsystem of elements by means of the four-microphone technique and use of the transfer function approach. Theoretical expressions are derived for the two source-location method, and these are shown to be identical to those for the two-load method. However, the two methods are conceptually different. The two-source method is shown to be functionally much more stable and entirely independent of the loading terminations. An uncertainty analysis is presented and used for estimation of possible nonequalization errors in the transfer matrix parameters. Finally, several examples of successful measurement use of the new method are included.

1. INTRODUCTION

In the past 15 years, the transfer matrix method has become very popular for analysis of one-dimensional acoustical systems. With acoustic pressure p and acoustic mass velocity v as the two state variables, the transfer matrix representation for a passive system (or subsystem) of Figure 1 is

$$\begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} p_2 \\ v_2 \end{bmatrix}, \tag{1}$$

where it is understood that the source is on the left side so that the forward progressive wave moves from left to right. Implicitly, the description is in the frequency domain; i.e., all state variables are in general complex functions of the forcing circular frequency ω, and so are the four-pole parameters A, B, C and D.

The theoretically predicted values of the four-pole parameters have often been verified indirectly: i.e., by measuring the overall performance of a system and comparing it with

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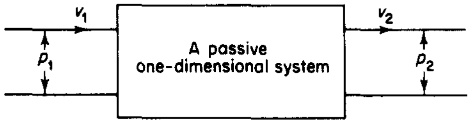

Figure 1. Four-pole representation of an acoustic system.

that predicted by means of successive multiplication of the theoretically predicted transfer matrices of the constituent elements of the system under examination. Perhaps the first direct verification was attempted by To and Doige [1, 2] making use of their transient testing method, for a stationary medium.

A moving medium makes measurements more difficult, inasmuch as the convective and dissipative effects of the mean flow introduce certain instabilities particularly at sudden area discontinuities. The transient testing technique was successfully extended to the direct measurement of the four-pole parameters of uniform tubes, flare tubes and sudden area changes, by Lung and Doige [3], for small mean flow velocities. They made use of a two-load, four-microphone technique.

The problems associated with measurements with a moving medium are accentuated with higher mean flow velocities which may appear at the throats of orifice plates and valves. Here, considerable flow noise is generated that interferes directly with measurements, making them unreliable or uncertain [4, 5]. This generative effect of mean flow is a relatively low-frequency phenomenon. The flow noise thus produced, being random in nature, makes it very difficult to employ random excitation that is generally used with the four-microphone two-load method, particularly at low frequencies that characterize pressure pulsations in gas pipelines. The two-load method suffers from an additional disadvantage in that the two loads may not be "sufficiently" different at all frequencies of interest.

It was in this context that an investigation was undertaken for development of a new measurement technique for the determination of transfer matrices of pipeline elements with flow. This resulted in a new method, called the two source-location method, to replace the two-load method [1-3], and the use of a signal enhancement technique (ensemble averaging in the time domain) to filter out the flow noise generated by the element under investigation. The new method has been tried successfully on orifice plates for typical mean flow Mach numbers. Incidentally, there does exist an earlier example in the literature where two sources are used for testing four-poles [6]. However, for the case of aeroacoustics the two source-location method is novel. The theory of the new method is described in the following sections. Comparisons with the two-load method are provided where applicable.

2. THEORY OF THE SOURCE-LOCATION METHOD

The method consists of the following: (a) measuring acoustic pressures (or rather their ratios called transfer functions) at four fixed locations, two upstream and two downstream of the test element, as shown in Figure 2(a), with a pseudo-random source on the left side; (b) shifting the source to the right side, as shown in Figure 2(b), and measuring acoustic pressures at the same four locations; (c) calculating A, B, C and D (the four-pole parameters of the test element) by means of a dual-channel FFT analyzer and use of the time-domain ensemble averaging and certain relations that are derived in what follows.

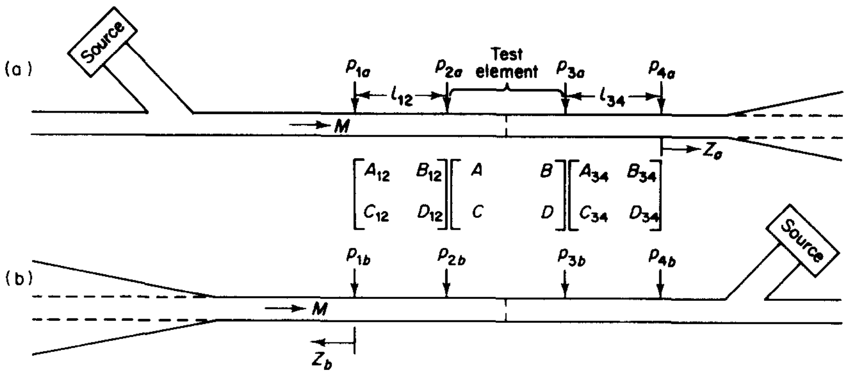

Figure 2. The two test configurations for the two source-location method.

The state variables at various junctions of Figure 2(a) are related as follows:

$$\begin{bmatrix} p_{1a} \\ v_{1a} \end{bmatrix} = \begin{bmatrix} A_{12} & B_{12} \\ C_{12} & D_{12} \end{bmatrix} \begin{array}{c} p_{2a} \\ \vdots \\ v_{2a} \end{array} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{array}{c} p_{3a} \\ \vdots \\ v_{3a} \end{array} \begin{bmatrix} A_{34} & B_{34} \\ C_{34} & D_{34} \end{bmatrix} \begin{bmatrix} p_{4a} \\ p_{4a}/Z_a \end{bmatrix}. \tag{2}$$

Thus

$$p_{3a}/p_{4a} = A_{34} + B_{34}/Z_a, \tag{3}$$

$$\frac{p_{2a}}{p_{4a}} = AA_{34} + BC_{34} + \frac{AB_{34} + BD_{34}}{Z_a} = A\left\{\frac{p_{3a}}{p_{4a}}\right\} + B\left\{C_{34} + \frac{D_{34}}{Z_a}\right\}$$

$$\frac{p_{1a}}{p_{4a}} = A_{12}(AA_{34} + BC_{34}) + B_{12}(CA_{34} + DC_{34}) + \frac{A_{12}(AB_{34} + BD_{34}) + B_{12}(CB_{34} + DD_{34})}{Z_a}, \tag{4}$$

or, upon making use of equations (3) and (4),

$$\frac{p_{1a}}{p_{4a}} = A_{12}\left\{\frac{p_{2a}}{p_{4a}}\right\} + B_{12}\left\{C\frac{p_{3a}}{p_{4a}} + D\left(C_{34} + \frac{D_{34}}{Z_a}\right)\right\}. \tag{5}$$

Before seeking similar relations for the pressures of Figure 2(b), one must note that the direction of the forward progressive wave and hence of the mass velocity v is from right to left in Figure 2(b). With reference to Figure 1, it is evident that equation (1) is to be replaced by

$$\begin{bmatrix} p_2 \\ -v_2 \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} D & B \\ C & A \end{bmatrix}\begin{bmatrix} p_1 \\ -v_1 \end{bmatrix}, \tag{6}$$

where $\Delta = AD - BC$ is the determinant of the matrix.

Applying this to the transfer matrices of all the three elements in Figure 2(b), and proceeding from the source to the load (impedance Z_b), one obtains

$$\begin{bmatrix} p_{4b} \\ v_{4b} \end{bmatrix} = \begin{bmatrix} \dfrac{D_{34}}{\Delta_{34}} & \dfrac{B_{34}}{\Delta_{34}} \\ \dfrac{C_{34}}{\Delta_{34}} & \dfrac{A_{34}}{\Delta_{34}} \end{bmatrix} \begin{array}{c} p_{3b} \\ \vdots \\ v_{3b} \end{array} \begin{bmatrix} \dfrac{D}{\Delta} & \dfrac{B}{\Delta} \\ \dfrac{C}{\Delta} & \dfrac{A}{\Delta} \end{bmatrix} \begin{array}{c} p_{2b} \\ \vdots \\ v_{2b} \end{array} \begin{bmatrix} \dfrac{D_{12}}{\Delta_{12}} & \dfrac{B_{12}}{\Delta_{12}} \\ \dfrac{C_{12}}{\Delta_{12}} & \dfrac{A_{12}}{\Delta_{12}} \end{bmatrix} \begin{bmatrix} p_{1b} \\ p_{1b}/Z_b \end{bmatrix}. \tag{7}$$

Thus

$$p_{2b}/p_{1b}=(1/\Delta_{12})\{D_{12}+B_{12}/Z_b\}, \tag{8}$$

$$\frac{p_{3b}}{p_{1b}}=\frac{1}{\Delta\Delta_{12}}\left\{DD_{12}+BC_{12}+\frac{DB_{12}+BA_{12}}{Z_b}\right\}=\frac{1}{\Delta}\left\{D\frac{p_{2b}}{p_{1b}}+\frac{B}{\Delta_{12}}\left(C_{12}+\frac{A_{12}}{Z_b}\right)\right\},$$

$$\frac{p_{4b}}{p_{1b}}=\frac{1}{\Delta_{34}\Delta\Delta_{12}}\left\{D_{34}(DD_{12}+BC_{12})+B_{34}(CD_{12}+AC_{12})\right.$$
$$\left.+\frac{D_{34}(DB_{12}+BA_{12})+B_{34}(CB_{12}+AA_{12})}{Z_b}\right\}, \tag{9}$$

or, upon making use of equations (8) and (9),

$$\frac{p_{4b}}{p_{1b}}=\frac{1}{\Delta_{34}}\left[D_{34}\frac{p_{3b}}{p_{1b}}+\frac{B_{34}}{\Delta}\left\{C\frac{p_{2b}}{p_{1b}}+\frac{A}{\Delta_{12}}\left(C_{12}+\frac{A_{12}}{Z_b}\right)\right\}\right]. \tag{10}$$

Incidentally, equations (3) and (8) yield expressions for the load impedances Z_a and Z_b: respectively,

$$Z_a=\frac{B_{34}}{(p_{3a}/p_{4a})-A_{34}}, \qquad Z_b=\frac{B_{12}}{\Delta_{12}(p_{2b}/p_{1b})-D_{12}}. \tag{11, 12}$$

Equations (4), (5), (9) and (10) may be solved simultaneously to evaluate A, B, C and D. Here it may be noted that

$$\Delta_{12}=A_{12}D_{12}-B_{12}C_{12}, \qquad \Delta=AD-BC, \qquad \Delta_{34}=A_{34}D_{34}-B_{34}C_{34}. \tag{13–15}$$

While Δ_{12} and Δ_{34} would be known *a priori*, Δ would not be. Equation (14) should therefore be used to write Δ explicitly in terms of A, B, C and D before attempting simultaneous solution of equations (4), (5), (9) and (10), with Z_a and Z_b given by equations (11) and (12).

For convenience, one can define a transfer function H_{ij} as

$$H_{ij}\equiv p_i/p_j, \tag{16}$$

so that

$$p_{ia}/p_{ja}\equiv H_{ij,a} \qquad \text{and} \qquad p_{ib}/p_{jb}\equiv H_{ij,b}. \tag{17}$$

Substituting the value of Z_a from equation (11) in equations (4) and (5), and rearranging, yields

$$(H_{34,a})A+\{C_{34}+(D_{34}/B_{34})(H_{34,a}-A_{34})\}B=H_{24,a} \tag{18}$$

and

$$(H_{34,a})C+\{C_{34}+(D_{34}/B_{34})(H_{34,a}-A_{34})\}D=(H_{14,a}-A_{12}H_{24,a})/B_{12}, \tag{19}$$

respectively. Similarly, substituting the value of Z_b from equation (12) in equations (9) and (10), and rearranging, yields

$$\{C_{12}+(A_{12}/B_{12})(\Delta_{12}H_{21,b}-D_{12})\}B+(\Delta_{12}H_{21,b})D=(H_{31,b}\Delta_{12})\Delta \tag{20}$$

and

$$\{C_{12}+(A_{12}/B_{12})(\Delta_{12}H_{21,b}-D_{12})\}A+(\Delta_{12}H_{21,b})C=\{(H_{41,b}\Delta_{34}-H_{31,b}D_{34})/B_{34}\}\Delta_{12}. \tag{21}$$

Elimination of Δ from equations (20) and (21) yields a single linear equation in A, B, C and D, which can be combined with equations (18) and (19) to obtain expressions for three of the four variables in terms of the fourth. Substitution of these expressions in equation (18), or (19), yields the value of this fourth variable and thence all the other three variables. These algebraic manipulations (omitted here) lead to the final expressions, as follows:

$$A=\frac{\Delta_{34}(H_{23,a}H_{43,b}-H_{23,b}H_{43,a})+D_{34}(H_{23,b}-H_{23,a})}{\Delta_{34}(H_{43,b}-H_{43,a})}, \tag{22}$$

$$B=B_{34}(H_{23,a}-H_{23,b})/\Delta_{34}(H_{43,b}-H_{43,a}), \tag{23}$$

$$C=\frac{(H_{13,a}-A_{12}H_{23,a})(\Delta_{34}H_{43,b}-D_{34})-(H_{13,b}-A_{12}H_{23,b})(\Delta_{34}H_{43,a}-D_{34})}{B_{12}\Delta_{34}(H_{43,b}-H_{43,a})}, \tag{24}$$

$$D=B_{34}\{(H_{13,a}-H_{13,b})+A_{12}(H_{23,b}-H_{23,a})\}/B_{12}\Delta_{34}(H_{43,b}-H_{43,a}). \tag{25}$$

The determinant is given by

$$\Delta=B_{34}(H_{13,a}H_{23,b}-H_{13,b}H_{23,a})/B_{12}\Delta_{34}(H_{43,b}-H_{43,a}). \tag{26}$$

In these final expressions [12],

$$\begin{bmatrix} A_{12} & B_{12} \\ C_{12} & D_{12} \end{bmatrix} = \mathrm{e}^{-M\beta_{12}} \begin{bmatrix} \cosh\beta_{12} & Y\sinh\beta_{12} \\ \sinh\beta_{12}/Y & \cosh\beta_{12} \end{bmatrix}, \qquad \Delta_{12}=\mathrm{e}^{-2M\beta_{12}},$$

$$\begin{bmatrix} A_{34} & B_{34} \\ C_{34} & D_{34} \end{bmatrix} = \mathrm{e}^{-M\beta_{34}} \begin{bmatrix} \cosh\beta_{34} & Y\sinh\beta_{34} \\ \sinh\beta_{34}/Y & \cosh\beta_{34} \end{bmatrix}, \qquad \Delta_{34}=\mathrm{e}^{-2M\beta_{34}},$$

$$\beta_{12}=(\mathrm{j}k_c+\alpha_c)l_{12}, \qquad \beta_{34}=(\mathrm{j}k_c+\alpha_c)l_{34}, \qquad k_c=k/(1-M^2), \qquad \alpha_c=\alpha/(1-M^2),$$

$$k=k_0+\alpha, \qquad \alpha=\alpha_0+MF/2D, \qquad Y=Y_0\{1-(\alpha/k_0)+\mathrm{j}\alpha/k_0\}.$$

Here α_0, M, F and D are the viscothermal pressure attenuation constant, the mean flow Mach number, Froude's friction factor and the pipe diameter, respectively.

3. THEORY OF THE TWO-LOAD METHOD

The two-load method consists of conducting the test with two different loads with the source on the same side. These two configurations are shown in Figure 3. The first configuration of Figure 3 is of course the same as in Figure 2. Therefore, equations (18) and (19) hold for this as well. Equations (20) and (21) will now be replaced by equations

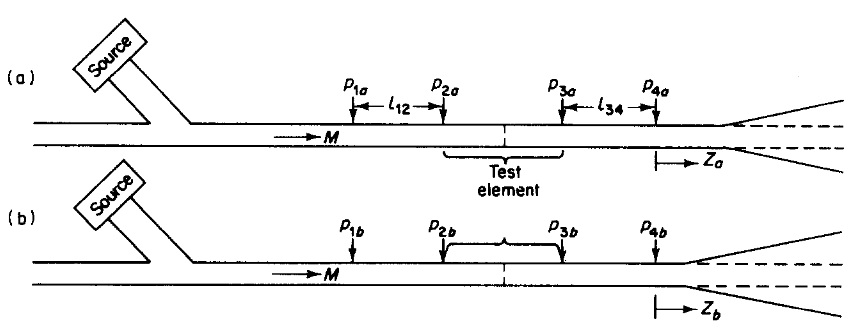

Figure 3. The two test configurations for the two-load method.

that are identical to equations (18) and (19) except that the subscript a is replaced by b. Simultaneous solution of these four equations reveals that equations (22)–(25) hold for the two-load method as well as the two-source method. Incidentally, the same expressions can also be obtained by rearranging those of reference [3]. This is not surprising inasmuch as the measurement of the four pole parameters should be independent of the methods one chooses to create the two test states. Significantly, however, the two source-location method is superior to the two-load method in that the former always creates two independent states, as is shown below.

4. FUNCTIONAL DIFFERENCE BETWEEN THE TWO METHODS

A comparison of equations (2) and (7) and of Figures 2 and 3, reveals that (i) while Z_a is downstream of microphone location 4, Z_b is downstream of location 1; (ii) all three matrices are reversed in the second configuration, as in equation (1) and (6); (iii) points 1 and 4 are interchanged, and so are points 2 and 3. Functionally, the two source-location method is much more reliable than the two-load method inasmuch as the former can work practically with any combination of terminal (or load) impedances Z_a and Z_b, whereas the latter would blow up as Z_b approaches Z_a. In the two-load method (see Figure 3), if Z_b tends to Z_a at any frequency, there would be no difference whatsoever between the two configurations, so that $H_{23,b} \to H_{23,a}$, $H_{43,b} \to H_{43,a}$ and $H_{13,b} \to H_{13,a}$, and substituting these limiting equalities in equations (22)–(25) indicates that A, B, C and D would then become indeterminate. In fact, as is shown later in this paper, even in the neighbourhood of these frequencies, the uncertainty of the two-load method is much greater than that of the two-source method, which incidentally would not fail even in the hypothetical eventuality of the two impedances Z_a and Z_b being identical.

In order to demonstrate the stability of the two-source method analytically, let (a) the test element be a uniform pipe of length l, and of the same diameter as the tube to the left and to the right of it, (b) the medium be inviscid and stationary, and (c) the terminations be anechoic in both the cases: i.e., $Z_a = Z_b = Y_0$, where Y_0 is the characteristic impedance. Then

$$\begin{bmatrix} A_{12} & B_{12} \\ C_{12} & D_{12} \end{bmatrix} = \begin{bmatrix} \cos k_0 l_{12} & \mathrm{j}\, Y_0 \sin k_0 l_{12} \\ (\mathrm{j}/Y_0) \sin k_0 l_{12} & \cos k_0 l_{12} \end{bmatrix}, \qquad \Delta_{12} = 1,$$

$$\begin{bmatrix} A_{34} & B_{34} \\ C_{34} & D_{34} \end{bmatrix} = \begin{bmatrix} \cos k_0 l_{34} & \mathrm{j}\, Y_0 \sin k_0 l_{34} \\ (\mathrm{j}/Y_0) \sin k_0 l_{34} & \cos k_0 l_{34} \end{bmatrix}, \qquad \Delta_{34} = 1,$$

$$H_{13,a} = p_{1a}/p_{3a} = \mathrm{e}^{\mathrm{j}k_0(l_{12}+l)}, \qquad H_{23,a} = p_{2a}/p_{3a} = \mathrm{e}^{\mathrm{j}k_0 l}, \qquad H_{43,a} = p_{4a}/p_{3a} = \mathrm{e}^{-\mathrm{j}k_0 l_{34}},$$

$$H_{13,b} = p_{1b}/p_{3b} = \mathrm{e}^{-\mathrm{j}k_0(l_{12}+l)}, \qquad H_{23,b} = p_{2b}/p_{3b} = \mathrm{e}^{-\mathrm{j}k_0 l}, \qquad H_{43,b} = p_{4b}/p_{3b} = \mathrm{e}^{+\mathrm{j}k_0 l_{34}}.$$

Incidentally, it may be noted that all the transfer functions in the b-configuration are reciprocals of the corresponding functions in the a-configuration, but these would be equal in the two-load method (!). Herein indeed lies the relative strength of the two source-location method. Nevertheless, making the foregoing substitutions in equations (22)–(25) yields (after some algebraic simplifications)

$$A = 2\mathrm{j} \cos k_0 l \sin k_0 l_{34} / 2\mathrm{j} \sin k_0 l_{34} = \cos k_0 l,$$

$$B = \mathrm{j}\, Y_0 \sin k_0 l_{34} (2\mathrm{j} \sin k_0 l)/2\mathrm{j} \sin k_0 l_{34} = \mathrm{j}\, Y_0 \sin k_0 l,$$

$$C = (-2\mathrm{j} \sin k_0 l_{12} \sin k_0 l_{34} \sin k_0 l)/(-2\, Y_0 \sin k_0 l_{12} \sin k_0 l_{34}) = (\mathrm{j}/Y_0) \sin k_0 l,$$

$$D = \mathrm{j}\, Y_0 \sin k_0 l_{34} \{2\mathrm{j} \sin k_0 l_{12} \cos k_0 l\}/(-2\, Y_0 \sin k_0 l_{12} \sin k_0 l_{34}) = \cos k_0 l.$$

Thus, not only the correct values of A, B, C and D are recovered but also it is significant that the denominators would be far from zero except at the frequencies that make $k_0 l_{12}$ and/or $k_0 l_{34}$ equal to $n\pi$, where n is a positive integer. This is a well-known fact from the theory of plane wave decomposition in a straight duct from two pressure measurements [7-11]. The validity of the measurements would extend over the full range if l_{12} and l_{34} were less than $\pi/3{\cdot}68$ times D. This is because one is making use of the plane wave theory which breaks down as $k_0 D/2$ approaches 1·84. In fact, the three-dimensional effects become significant at still lower frequencies [12] and therefore, reasonable design values of l_{12} and l_{34} are given by

$$l_{12}, l_{34} < 0{\cdot}75\ D. \tag{27}$$

Should one use l_{12} and/or l_{34} larger than D, one should keep the highest frequency proportionately down: i.e., the highest frequency should not be decided by the plane wave propagation inequality

$$k_0 D/2 < 1{\cdot}84, \tag{28}$$

but by the reliability inequality [10, 11]

$$0{\cdot}1\pi < k_0 l_{12},\ k_0 l_{34} < 0{\cdot}8\pi. \tag{29}$$

5. UNCERTAINTY ANALYSIS OF THE TWO METHODS

Expressed in terms of the measured pressure transfer functions, A, B, C and D of the test element are governed by the same equations (22)-(25) for the two source-location method (Figure 2) and the two-load method (Figure 3). Thus, for both the methods, the same expressions would hold for estimation of uncertainty in any of the four-pole parameters on account of a possible error in prediction or measurement of the velocity of wave propagation a_0 (and hence k_0), the pressure attenuation coefficient α, the lengths l_{12} and l_{34}, and the magnitude or phase of any of the measured transfer functions (resulting from the equalization errors or the data-processing errors).

The general principles of error analysis are well known [9-11]. Uncertainty in any of the four-pole parameters, say A, due to a possible error in one of the independent variables, say x, may be evaluated directly as

$$(\delta A)_x = A(x+\delta x) - A(x), \tag{30}$$

or, by making use of the Taylor series expansion and retaining only the first order term,

$$(\delta A)_x = (\partial A/\partial x)\delta x. \tag{31}$$

Either of these two expressions may be used with ease on the explicit expressions (22)-(25) for A, B, C and D. Equation (31) gives a little more insight into the relative significance of the various independent variables. It is obvious that uncertainty in A would shoot up (or the calculated value of A would become highly unreliable) at frequencies that make the denominator of A tend to zero. This, as pointed out in the preceding section, is what makes the two-load method highly unreliable at frequencies at which Z_b is nearly equal to Z_a because then $H_{43,b}$ would be nearly equal to $H_{43,a}$ making the denominator of all of the four-pole parameters very small.

6. ESTIMATION OF NON-EQUALIZATION ERROR

A transfer function H_{ij}, being defined as ratio of pressure at the ith junction to that at the jth junction (or microphone location), could be in error because of the possibly unequal frequency response of the two channels (each consisting of the microphone cavity, mirophone, preamplifier, a cable, measuring amplifier, another cable, and the input amplifier of the dual-channel FFT analyzer). In other words, if both the microphones were subjected to the same sound field, the ratio of the measured pressures would in general be not equal to unity but

$$1+\varepsilon(\omega, M), \tag{32}$$

where $\varepsilon(\omega, M)$ is the non-equalization error (negative of the equalization correction). This would be a function of the excitation frequency ω, and the mean flow Mach number, M. The dependence on M could be eliminated if the size of the microphone cavity could be made exactly equal for both the channels; or, in other words, if all the microphone holders could be fabricated with the same dimensions and very close tolerances.

Suppose that the first channel is used for the numerator pressure p_i and the second for the denominator pressure p_j in evaluating the transfer function: i.e.,

$$H_{ij} = p_i/p_j. \tag{33}$$

Then, every transfer function would be subject to the same non-equalization error $\varepsilon(\omega, M)$, and therefore would become multiplied by the same factor (32)†. Thus, uncertainties in A, B, C and D due to the non-equalization error $\varepsilon(\omega, M)$ may be calculated readily as follows:

$$(\delta A)_H = \frac{\Delta_{34}(H_{23,a}H_{43,b}H_{23,b}H_{43,a})(1+\varepsilon)^2 + D_{34}(H_{23,b}-H_{23,a})(1+\varepsilon)}{\Delta_{34}(H_{43,b}-H_{43,a})(1+\varepsilon)} - \frac{\Delta_{34}(H_{23,a}H_{43,b}-H_{23,b}H_{43,a}) + D_{34}(H_{23,b}-H_{23,a})}{\Delta_{34}(H_{43,b}-H_{43,a})}, \tag{34}$$

or

$$(\delta A)_H = \{(H_{23,a}H_{43,b}-H_{23,b}H_{43,a})/(H_{43,b}-H_{43,a})\}\varepsilon(\omega, M). \tag{35}$$

Similarly, it may be verified that

$$(\delta B)_H = 0, \qquad (\delta C)_H = \left\{\frac{(H_{13,a}-A_{12}H_{23,a})H_{43,b}-(H_{13,b}-A_{12}H_{23,b})H_{43,a}}{B_{12}(H_{43,b}-H_{43,a})}\right\}\varepsilon(\omega, M), \tag{36, 37}$$

$$(\delta D)_H = 0. \tag{38}$$

Incidentally, it may noted from equations (22)-(25) and (35)-(38) that uncertainty in the four-pole parameters due to the possible non-equalization error $\varepsilon(\omega, M)$ comes into play only because of the quadratic terms in the numerator: i.e., those terms in the numerator the degree of which does not match with that of the denominator. It may be noted that this type of error is a bias error, and can be eliminated by calibrating the measurement channels.

7. CONCLUDING REMARKS

The two source-location method described in the foregoing sections is similar to the two-load method inasmuch as the same expressions govern the four-pole parameters of the test element in either scheme. Significantly, however, the two source-location method is functionally much more stable than the two-load method. In fact, it is entirely independent of the loading terminations on either side. This has been amply demonstrated for

† Here it may be noted that H_{ij} would also be subject to random errors and bias errors [9-11]. However, as one is assumed to be using a pseudo-random signal and time-averaging for signal enhancement, these errors would be minimal.

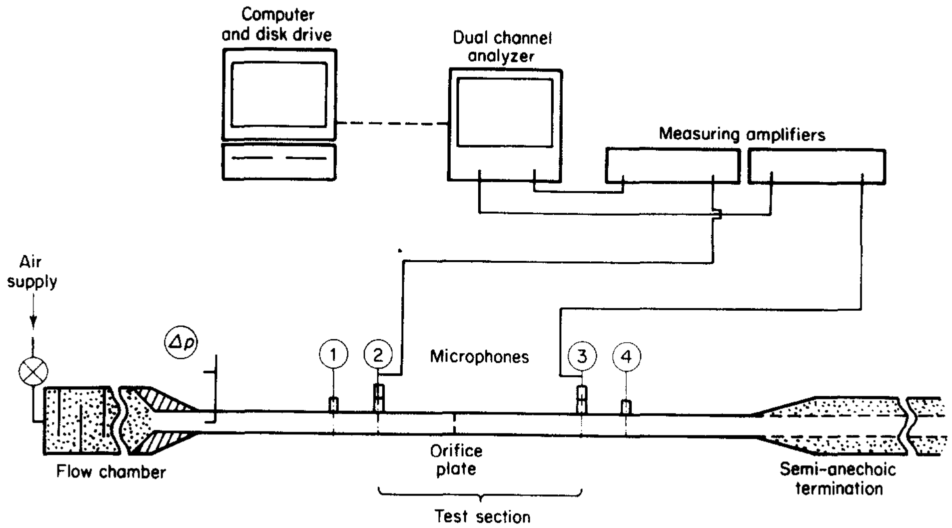

Figure 4. Basic arrangement for laboratory tests.

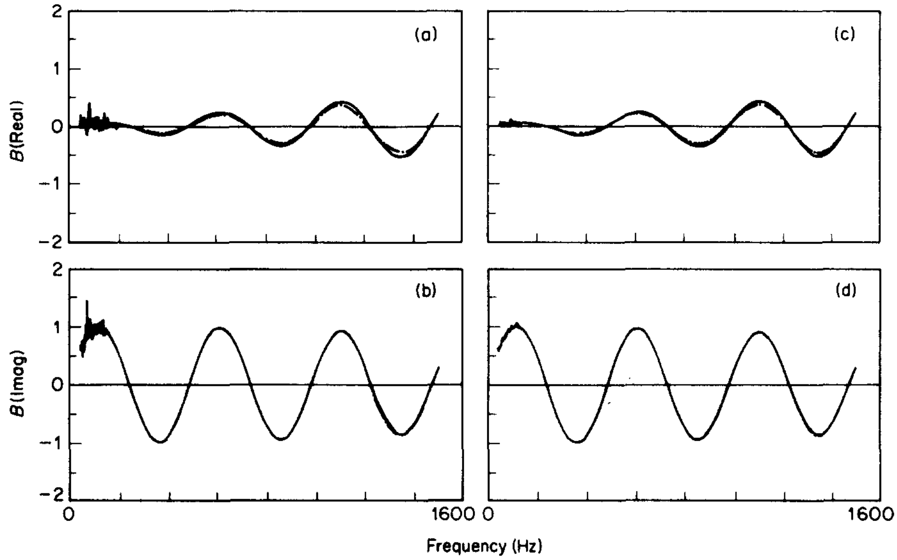

Figure 5. B parameter for a straight pipe, $M = 0{\cdot}03$. (a) and (b), two load method; (c) and (d), two source-location method.

orifice plates as well as uniform tubes, with typical mean flow velocities: see Figures 4, 5 and 6 Details of the basic arrangement for laboratory tests (Figure 4) are given in references [13, 14]. The broken line curves in Figure 5 indicate the analytically computed values [12]. Figures 5 and 6 are two of the scores of graphs given in reference [13], representing successful measurement when the two source-location method is used and the superiority thereof over the two-load method [13].

ACKNOWLEDGMENTS

The first author would like to thank the authorities of the Indian Institute of Science for the sabbatical leave that made this research work possible. Financial support from

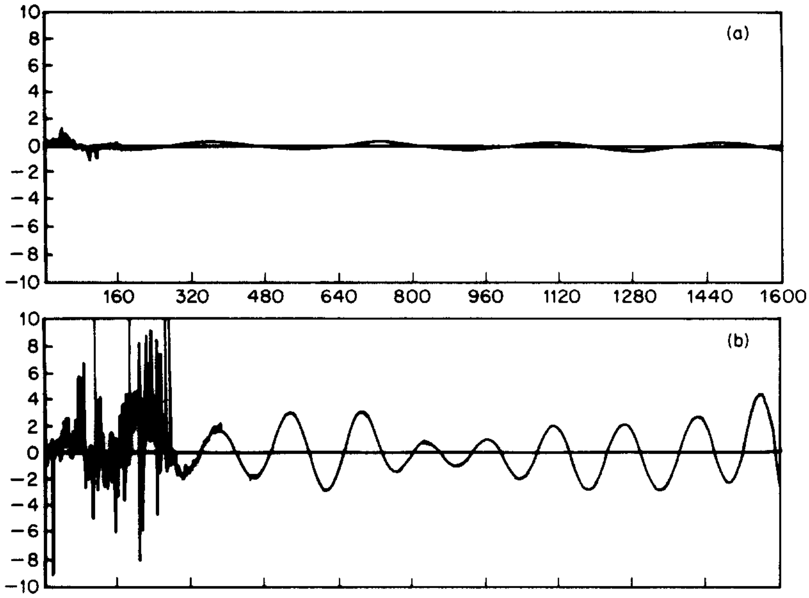

Figure 6. Percentages uncertainty in A due to a 1% error in $H_{23,b}$ (real part). (a) Two source-location method; (b) two-load method.

Nova Husky Research Corporation of Calgary and the National Science and Engineering Research Council of Canada is also gratefully acknowledged.

REFERENCES

1. C. W. S. To and A. G. Doige 1979 *Journal of Sound and Vibration* **62**, 207–222. A transient testing technique for the determination of matrix parameters of acoustic systems, I: theory and principles.
2. C. W. S. To and A. G. Doige 1979 *Journal of Sound and Vibration* **62**, 223–233. A transient testing technique for the determination of matrix parameters of acoustic systems, II: experimental procedures and results.
3. T. Y. Lung and A. G. Doige 1983 *Journal of the Acoustical Society of America* **73**, 867–876. A time-averaging transient testing method for acoustic properties of piping systems and mufflers with flow.
4. K. K. Botros, W. M. Jungowski, W. Studzinski and J. L. Szabo 1986 *International Gas Research Conference, Vol. I: Gas Distribution and Transmission, Toronto, September*, 243–256. Pressure pulsations at orifice plates and general pipeline flow-acoustic simulation.
5. W. M. Jungowski, K. K. Botros, W. Studzinski and J. L. Szabo 1986 *ASME Winter Annual Conference, Proceedings of the Symposium on Measuring and Metering of Unsteady Flow, Anaheim, December*, 21–31. Measuring the transmission characteristics of pulsating flows through orifice plates and a ball valve.
6. C. Fumoux, D. Leducq and G. Marquette 1985 *Proceesings INTER-NOISE* 85, 1239–1242. Transfer matrix measurement of hydraulic piping system elements.
7. J. Y. Chung and D. A. Blaser 1980 *Journal of the Acoustical Society of America* **69**, 907–921. Transfer function method of measuring in-duct acoustic properties: I, theory; II, experiment.
8. P. O. A. L. Davies, M. Bhattacharya and J. L. Bento-Coelho 1980 *Journal of Sound and Vibration* **72**, 539–542. Measurement of plane wave acoustic fields in flow ducts.
9. A. G. Seybert and B. Soenarko 1981 *Journal of the Acoustical Society of America* **69**, 1190–1199. Error analysis of spectral estimates with application to the measurement of acoustic parameters using random sound fields in ducts.
10. H. Bodén and M. Åbom 1986 *Journal of the Acoustical Society of America* **79**, 541–549. Influence of errors in the two-microphone method for measuring acoustic properties in ducts.

11. M. ÅBOM and H. BODÉN 1988 *Journal of the Acoustical Society of America* **83**, 2429–2438. Error analysis of two-microphone measurements in ducts with flow.
12. M. L. MUNJAL 1987 *Acoustics of Ducts and Mufflers.* New York: Wiley-Interscience. See pp. 121–123.
13. A. G. DOIGE, M. L. MUNJAL, H. S. ALVES and P. S. ALVES 1987 *Final Report of the NHRC project, University of Calgary, Canada.* Development of a new measurement technique for the determination of transfer matrices of pipeline element with flow.
14. A. G. DOIGE, M. L. MUNJAL and H. S. ALVES 1988 *Proceedings NOISE-CON* 88, 481–486. An improved experimental method for determining transfer matrices of pipeline elements with flow.