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# RESONANCE EFFECTS IN WAKE SHEDDING FROM PARALLEL PLATES: SOME EXPERIMENTAL OBSERVATIONS

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A series of tests with air flowing over a cascade of flat parallel plates has shown that pressure fluctuations occur with amplitudes of the same order of magnitude as the air stream dynamic head. The fluctuations form a sequence of resonances and are excited by wake shedding from the plate trailing edges.

The resonances are caused almost entirely by acoustic effects and have little or no relation to the mechanical vibration of the plates.

## 1. INTRODUCTION

There are many recorded cases where the shedding of periodic wakes (Karman vortices) from plates parallel to a fluid stream has been found to involve a series of distinct resonances. These are often referred to as "Aeolian tones" and the explanation commonly given is that the natural shedding frequency is close to the natural frequency of vibration of the plate concerned and so mechanical vibration occurs. When resonance occurs, the vortices are considered to be shed at the natural frequency of vibration, which is invariably at or below the natural frequency of shedding from a rigid plate.

Forster (1) has recently reported a case where resonances occurred with air flowing over six radial spokes supporting the bullet of an air flow model of the casings of the Hinkley Point main gas circulators. The spokes were located at a position of high velocity, closely following the intake acceleration and at the beginning of the diffuser, the annulus of the model must have been about 9 in. inside diameter and 16 in. outside diameter at this point (see Figure 3 of reference 1).

By considering the possibility that the resonances are purely acoustic and taking the simple modes described by Forster, resonant frequencies can be predicted at 1060, 1980 and 2860 c/s for 6, 12 and 18 radial nodes respectively (assuming the velocity of sound to be 1120 ft/sec); the corresponding observed values are 980, 1830 and 2600 c/s. The ratios between the frequencies of the three modes are 1/1·875/2·70 as predicted and the observed values are all between 7·5 and 9% below the predicted values. It is unlikely that any mechanical effects would have the same ratios between three easily excited modes and it would appear that the resonances are acoustic rather than mechanical.

This paper is an account of a preliminary investigation to establish whether or not acoustic resonances can be a major effect in connection with wake shedding. The results raise as many questions as answers but the importance of acoustic effects is clearly shown.

## 2. TESTS

A series of tests have been done using a low speed wind tunnel at the University College of Swansea with a working section of octagonal cross-section, 24 in. across flats. In the belief that it would be a simple case to analyse, a sequence of flat parallel plates was supported in two vertical side members spanning the tunnel as shown in Plate 1. Equally spaced slots were provided in the side members so that various plate configurations could be used and plates were provided as shown in the following table.

TABLE I

| Chord (in.) | Thickness (in.) | Materials |
|---|---|---|
| 3 | $\frac{1}{16}$ | Light alloy |
| 1 | $\frac{1}{16}$ | Light alloy |
| 3 | $\frac{1}{16}$ | Brass |
| 3 | $\frac{3}{32}$ | Brass |
| 0·0625 dia. rods. | | Brass |

The span (measured between the inside surfaces of the side walls) was 4 in., and the thicker brass plates were machined down at the ends to fit the $\frac{1}{16}$ in. slots in the side walls.

The materials were chosen to give a significant change in the ratio $E/\rho$; thus for given plate dimensions, the natural frequency of mechanical vibration in any mode for the light alloy plate will be 40–50% higher than for the brass plate.

A similar argument applies with changes in the thickness of the brass plates from $\frac{1}{16}$ to $\frac{3}{32}$ in., as the natural frequency for any mode is proportional to the thickness while the natural wake shedding frequency at a given velocity is, approximately, inversely proportional to the thickness.

The air speed was measured using the normal tunnel pressure connections and allowance made for the blockage of the test rig (7% of the tunnel area). The velocities are therefore probably only accurate in absolute values to say ± 5%, but any error will be consistent and will only affect conclusions regarding the Strouhal number for the plates. (Strouhal number $=ft/v$.)

The existence of many resonances was easily detected audibly and preliminary testing was done using nothing more than a stethoscope fitted with a length of hypodermic tubing and an oscillator and loudspeaker as a frequency reference. At a later stage two Brüel and Kjær half-inch microphones fitted with 4 mm probes were used in conjunction with a wave analyser and oscilloscope to obtain more accurate frequencies and to measure the sound pressure levels. Owing to the difficulties of working inside the wind tunnel it proved to be impossible to investigate the nodal configurations fully and the rig is at present being redesigned to allow more complete investigations by providing for comprehensive traversing of the microphones from outside the tunnel.

In the tests reported here one microphone was fixed with the tip of the probe about $\frac{1}{2}$ in. in from one side support and about 1 in. upstream of the leading edges of the plates when 3 in. plates were used. This microphone was used only as a phase reference when checking the nodal pattern and was not calibrated.

The second probe was damped in accordance with the maker's instructions and carefully calibrated using a $\frac{1}{4}$ in. Brüel and Kjær microphone as reference. A curve of sensitivity against frequency from 100 c/s to 6 kc/s was obtained and all subsequent readings of sound pressure level (SPL) corrected accordingly. This unit was mounted so that it could be traversed vertically some 8 in. around the mid-height of the tunnel, and horizontally (parallel to the plate trailing edges) from outside the side supports to just beyond the mid-span. All movement was in a plane about 1 in. downstream of the trailing edges of the plates.

The upstream microphone output was displayed on one trace of the oscilloscope and the time base synchronized to this trace. The downstream microphone was connected to the wave analyser and the output displayed on the other trace of the oscilloscope.

With each arrangement of plates tested, the tunnel speed was increased slowly from the minimum until a resonance was detected, either audibly or on the oscilloscope traces. When the speed was steady, the time base was adjusted to give a steady picture and locked to the reference signal. The downstream microphone was then moved vertically and horizontally until a maximum reading was obtained and the frequency and SPL found. In all cases the wave analyser was used at "maximum selectivity" for checking the frequencies. In this condition the response was 3dB down at ± 6% in frequency. When tuned to give the maximum reading, the analyser could in all cases be switched to "frequency selection off" with negligible effect on the SPL reading or the appearance of the oscilloscope trace, indicating that the signals were almost pure sine waves.

The microphone was then traversed systematically vertically and horizontally as far as possible and the positions of the nodes observed by noting the phase changes relative to the reference microphone. In almost all cases (see section 3.3) the signal decreased steadily to zero and then increased in the opposite phase as each node was passed, indicating the position of the node very clearly.

After this the tunnel speed was increased slowly, with one of three results.

(a) The signal disappeared. The speed was then increased further until another resonance was found, invariably at a higher frequency.

(b) The signal abruptly changed to a higher frequency. The speed was held at this value.

(c) The signal persisted at the same frequency, the speed was then held after a small increase in velocity had been made.

In all cases the complete procedure described above was repeated; where the nodal pattern could be clearly defined, a change of frequency was always found to coincide with a change in the mode while continuation at the same frequency involved no change in the mode.

A further series of tests was done, each with a single plate at the centre of the tunnel. In no case could resonances of any kind be detected. Measurements of frequency were obtained by placing the microphone probe just to one side of the wake of the plate where a maximum signal was obtained. In all cases the frequency increased progressively with the tunnel speed.

## 3. RESULTS

### 3.1. SINGLE PLATES

The results for single plates are shown in Figure 1 from which it is seen that the slope of the frequency vs. velocity line is nearly constant for each plate. The light alloy and brass plates $\frac{1}{16}$ in. thick × 3 in. chord both gave the same results and only one set is shown for clarity. Strouhal numbers (frequency × thickness/velocity) obtained from the mean lines are:

$\frac{1}{16}$ in. thick × 1 in. chord, 0·217;

$\frac{1}{16}$ in. thick × 3 in. chord, 0·195;

$\frac{3}{32}$ in. thick × 3 in. chord, 0·22.

In all cases the pressure fluctuations could only be detected in and near to the plate wakes, the maximum pressure amplitudes being 0·02 to 0·03 × $\frac{1}{2}\rho v^2$, or if a reference r.m.s. pressure of $\frac{1}{2}\rho v^2/\sqrt{2}$ is adopted, the maximum SPL's were 30 to 35 dB below the reference.

### 3.2. ROUND RODS

The test with a single rod produced the same type of results as the flat plates; i.e., a progressive increase of frequency with air speed with no detectable resonances. When a regular array of rods was tested exactly the same result was obtained, with no evidence of any type of resonance. The results have therefore been omitted from Figure 1 as the slope of this line is of no real interest.

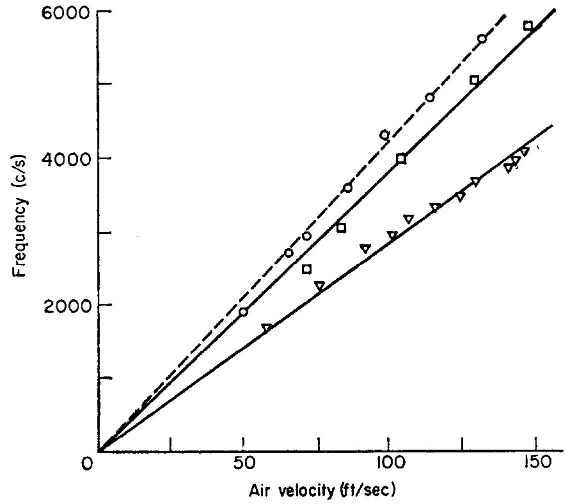

Figure 1. Frequency/velocity results for single plates (no resonances observed). —○———○——, $\frac{1}{16}$ in. × 1 in.; —□—□—, $\frac{1}{16}$ in. × 3 in.; —▽—▽—, $\frac{3}{32}$ in. × 3 in.

### 3.3. REGULARLY SPACED PLATES—QUANTITATIVE RESULTS

The results for the plate configurations which were examined most thoroughly are shown in Figures 2 to 6. In each case the plate arrangement is shown diagrammatically, the distances indicated being measured to the centre lines of the plates and the surfaces of the end walls. Where the nodal pattern could be clearly established notes are added accordingly, but it must be remembered that the exploration was confined to a plane approximately 1 in. downstream of the trailing edges of the plates. It is known that in some cases nodes occur between the plates in planes normal to the direction of flow, but, as it proved to be impossible to explore this fully in every case, no reference to such nodes is made on the figures. Such information on this as was obtained is given in section 3.4 below. The statement "nodes at plates" means therefore that nodes were observed downstream of the plates and it is assumed that the same result would be found if traverses were made closer to the trailing edges of the plates.

Where a particular frequency was observed over a small velocity range the result is given as a single point because the velocity range was too small to be measured properly. Where the velocity range was greater, points are shown for the conditions examined thoroughly and the points joined by straight lines.

The frequencies observed with each arrangement of plates may be compared with the frequency shed by the corresponding single plate, as the appropriate mean lines from Figure 1 have been superimposed.

The magnitude of the pressure fluctuation can be seen in relation to the air velocity by comparing them with the superimposed curve of SPL corresponding to the reference r.m.s. pressure of $\frac{1}{2}\rho v^2/\sqrt{2}$.

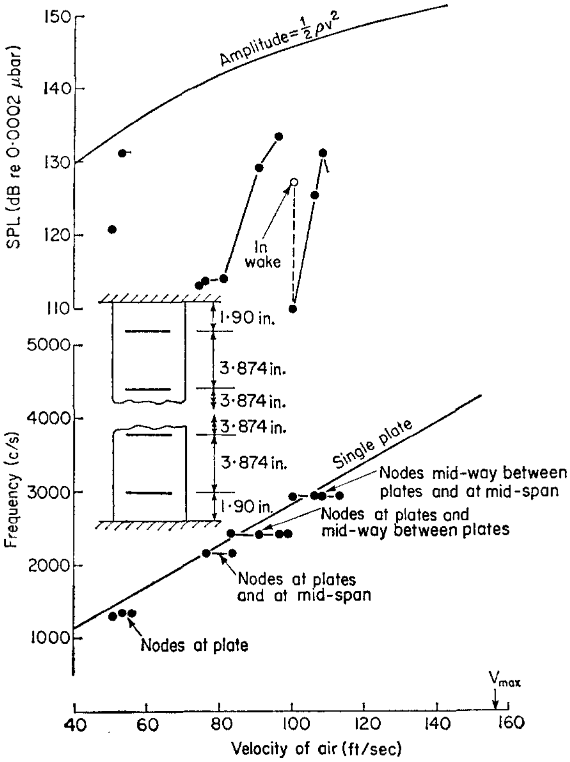

Figure 2. $\frac{1}{16}$ in. thick × 3 in. chord light alloy plates. Measurements taken 1 in. downstream of plate trailing edges.

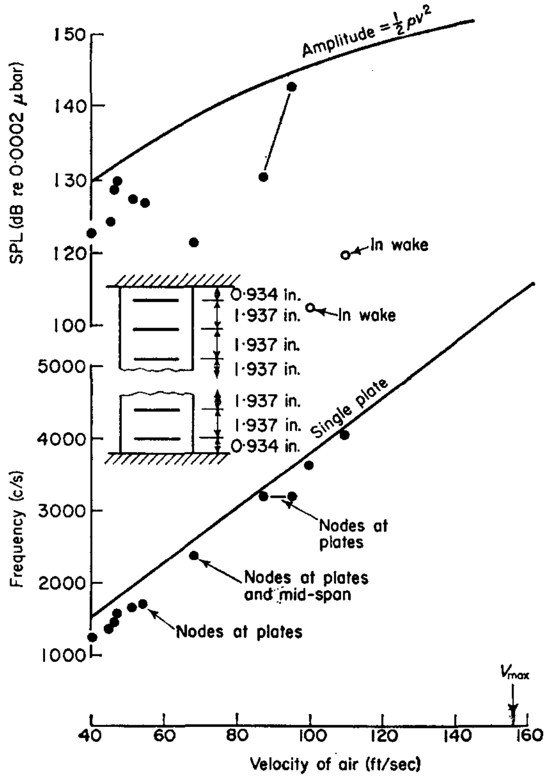

Figure 3. $\frac{1}{16}$ in. thick × 3 in. chord brass plates. Measurements taken 1 in. downstream of plate trailing edges.

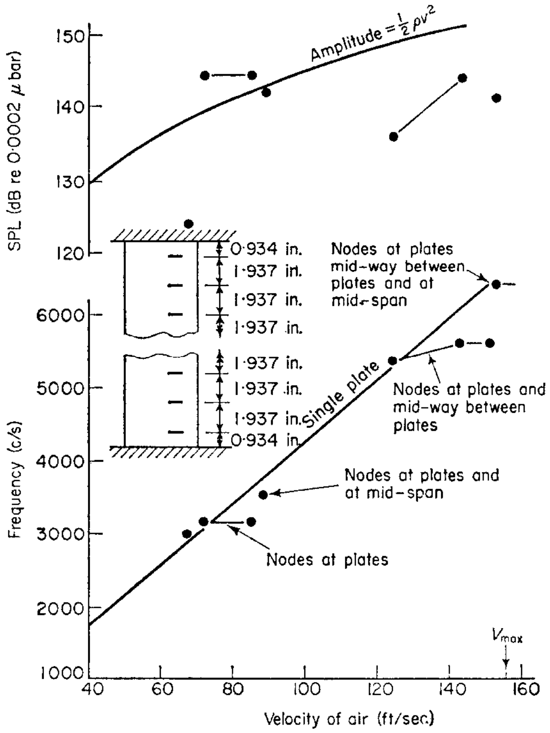

Figure 4. $\frac{1}{16}$ in. thick × 1 in. chord light alloy plates. Measurements taken 1 in. downstream of plate trailing edges.

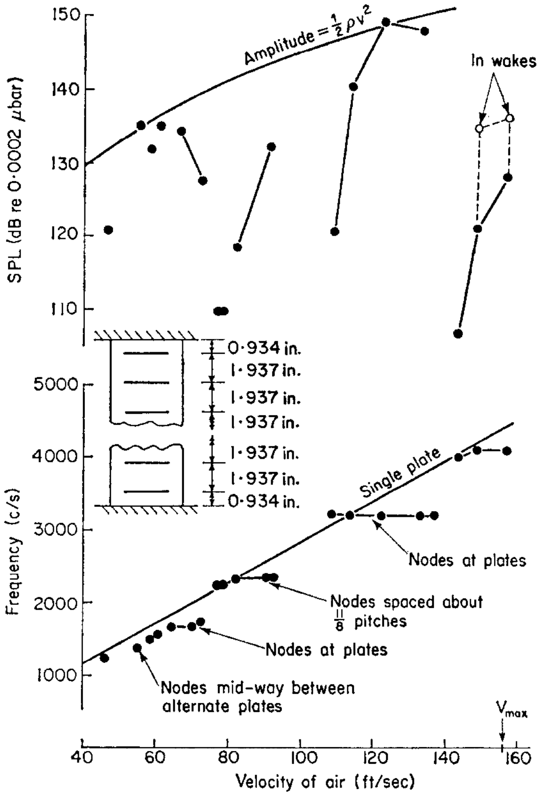

Figure 5. $\frac{3}{32}$ in. thick × 3 in. chord brass plates. Measurements taken 1 in. downstream of plate trailing edges.

All the results were obtained with increasing tunnel speed. Some checks were made with reducing speed and almost identical results obtained, with the onset and cessation of each mode of oscillation at slightly reduced velocities: i.e. there is a slight hysteresis effect with variation of speed.

At one point using $\frac{3}{32}$ in. brass plates (Figure 5) the usual clearly defined pattern was not found. A resonance occurred at velocities from 144 ft/sec to the maximum available at 156 ft/sec, the frequency changed slightly with increasing velocity and when the microphone was traversed across the wake itself an abrupt phase change was observed about the centre line. When traversing parallel to the plate trailing edge there was no variation over most of the span but a slight increase in levels close to the side supports, especially near the plates.

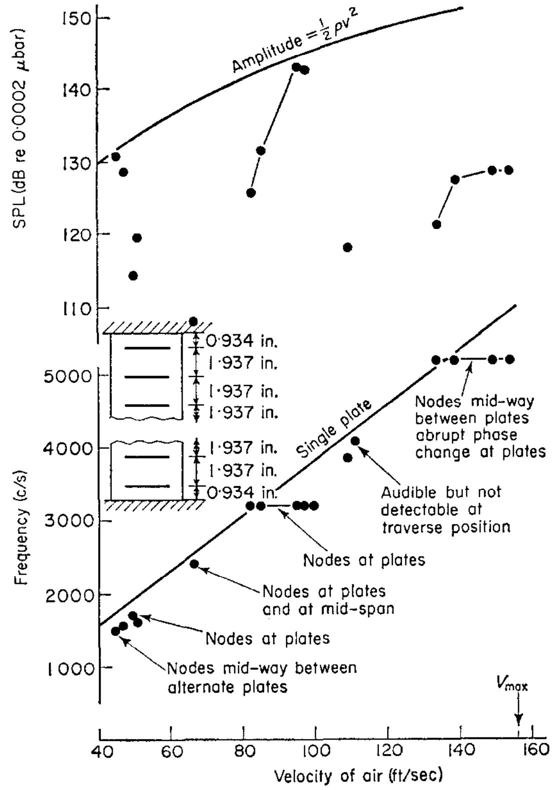

Figure 6. $\frac{3}{32}$ in. thick × 3 in. chord brass plates. Measurements taken 1 in. downstream of plate trailing edges.

Somewhat similar effects were observed at two points with $\frac{1}{16}$ in. × 3 in. brass plates (Figure 3) at 3650 and 4050 c/s and with $\frac{3}{32}$ in. × 3 in. brass plates at the wide spacing (Figure 6) at 2950 c/s. In the former the amplitudes were low and they occurred over very limited velocity ranges, in the latter case this effect occurred at 101 ft/sec but with increasing velocity the amplitude between the plates increased and clearly defined nodes appeared mid-way between the plates and at mid-span.

### 3.4. REGULARLY SPACED PLATES—QUALITATIVE OBSERVATIONS

A number of points are shown in the results for which no mode is defined. These occur at the lower frequencies and in general such nodes as could be reached within the microphone traversing range indicated that the spacing was not simply related to the plate spacings. They appear to divide the total span of the tunnel into regular patterns regardless of the plate positions.

In an attempt to explore these modes more fully a microphone was held by hand through a suitable hole in the tunnel wall and moved over the whole range, but the interference to the flow made this difficult and reliable statements of the mode corresponding to each frequency are not possible. However, the general conclusion was reached that the lower frequencies are related more to the tunnel geometry than the plate positions.

An attempt was also made to explore the three-dimensional conditions more fully with the hand-held microphone and the stethoscope. In most cases the disturbance to the flow caused changes in the mode (indicated by changes in frequency) as the microphone was moved about, but in one particular condition this did not happen. The high levels obtained at 3200 c/s when using 3 in. plates 1·937 in. apart (Figures 2, 3 and 5) were very stable and the microphone could be moved about freely. In this condition it was found that there was a node normal to the flow at about mid-chord and nodes in the planes of the plates extending upstream and downstream. The SPL measured between these nodes decreased gradually upstream and downstream but no further nodes were found. Large amplitudes were found at the plate surfaces around the $\frac{1}{4}$ and $\frac{3}{4}$ chord positions. This is illustrated in Figure 7.

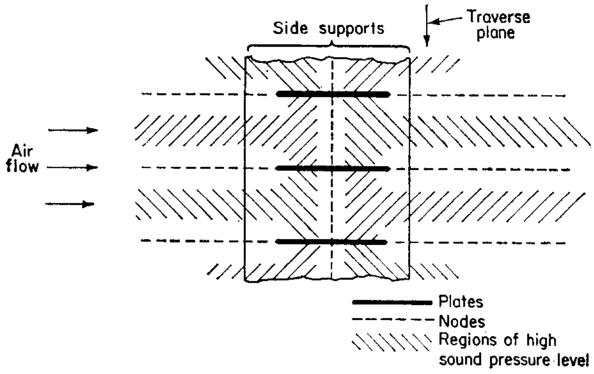

Figure 7. Diagrammatic representation of the mode at 3200 c/s with 3 in. chord plates 1·937 in. apart.

### 3·5. OTHER CONFIGURATIONS

A limited amount of testing was done at other plate spacings, as follows.

(a) Spacings here were generally as shown in Figures 2 to 6, but with the intervals between the last plate at each end and the solid wall doubled, i.e. practically the same as the spaces between adjacent plates. This change made no difference to the frequencies and the nodes but produced small changes in amplitudes, sometimes increases, sometimes reductions.

(b) $\frac{1}{16}$ in. × 3 in. plates were all spaced 0·934 in. apart (including the end intervals). This gave a great many more resonances, especially above 3000 c/s with a "silent" band between 2000 and 3000 c/s (64 to 87 ft/sec). In some cases two frequencies occurred together with resultant beating, it was decided that the complexity of this configuration did not justify further investigation until more complete traversing was possible.

(c) A few random arrangements of irregularly spaced plates were tested. The number of resonances was less than with regular spacing but, where resonances did occur, the amplitudes were much the same.

(d) The tunnel was run over the whole speed range with the supporting framework alone in position (i.e. with no plates of any kind). This confirmed that the supports, which had 45° re-entrant trailing edges and semicircular leading edges, did not shed any significant regular vortices. The only discrete tone detectable occurred solely at high speeds and was much lower in frequency than any of the results quoted above and of low amplitude.

## 4. DISCUSSION

### 4.1. MECHANICAL AND ACOUSTIC RESONANCES

By comparing Figures 2 and 3 (i.e., comparing light alloy with brass plates in the same configurations) a number of resonances were found in both cases, in particular:

TABLE 2

| Frequency (kc/s) | Brass plates | | Light alloy | |
|---|---|---|---|---|
| | Velocity | SPL (max.) | Velocity | SPL (max.) |
| 1·5 | 46 | 129 | 45 | 131 |
| 1·6 | 47·5 | 130 | 47·5 | 128·5 |
| 1·7 | 55 | 127 | 50 | 115 |
| 2·38 | 68 | 121·5 | 67 | 108 |
| 3·2 | 87 to 95 | 142·5 | 82 to 99 | 143 |

Where the mode was satisfactorily explored (i.e., at 1700 c/s and 3200 c/s) these were the same for brass and light alloy plates; for these frequencies therefore the change of material seems to have had no significant effect. However at 5200 c/s a resonance was found with the light alloy plates only; this was low in amplitude and the abrupt phase change at the plate does not fit into the general pattern. If this resonance was due to mechanical vibration the change from light alloy to brass should reduce its frequency to somewhere between 3500 and 3700 c/s. The results do in fact show a resonance at 3600 which was measurable in the wake only and of very low amplitude elsewhere.

There are then examples of both acoustic and mechanical resonances, the former giving several frequencies and producing some amplitudes only 2 or 3 dB down from the reference amplitude ($\pm \frac{1}{2}\rho v^2$), whereas the latter occurs only once at a level about 25 dB down.

Comparison of Figures 3 and 5 (i.e., of plates of the same material but of different thicknesses) shows that the majority of the resonances appear at the same frequencies but at different velocities, the difference being consistent with the change in the natural wake frequency. The amplitudes are similar in relation to the reference level at the appropriate velocities, with the thicker plate giving slightly higher relative values in most cases.

The weak resonances at 3600 and 4500 c/s for $\frac{1}{16}$ in. plates have the appearance of mechanical resonances in that amplitudes are much greater in the wakes than elsewhere. The same modes would be expected to have frequencies of 5400 and 6075 c/s, respectively in the thicker plates which, unfortunately, come outside the available velocity range of the tunnel.

The apparent mechanical resonance at 4100 c/s with $\frac{3}{32}$ in. plates would be expected to appear at about 2700 c/s with $\frac{1}{16}$ in. plates but, if it did exist, the amplitude must have been very low as it was not detected. When the spacing of the plates was doubled (compare Figures 5 and 6) the resonance at 4100 c/s again seems to have disappeared or become too weak to be detected.

Attempts to correlate the frequencies observed with the mode are in many cases hampered by lack of information on the exact mode. There is however one particular relationship which can be found in the results. There are a number of cases where a node was found at mid-span, suggesting a standing wave of $\lambda/2$ across the span combined with whatever other nodes occur. One would then expect to find a similar mode but omitting the node at mid-span where the frequencies are related by $f_2^2 = f_1^2 + f^2$, where $f_1$ is the lower observed frequency, $f_2$ is the frequency with a node at mid-span and $f$ is the frequency of a wave of half wavelength equal to the span (i.e., $f = 1120/2 \times 4/12) = 1680$ c/s, the span being 4 in.).

Four such pairs can be found:

(a) $\frac{1}{16}$ in. × 3 in. plates, both light alloy and brass (Figures 2 and 3), $f_1 = 1700$, $f_2 = 2380$;

(b) $\frac{1}{16}$ in. × 1 in. plates (Figure 4), $f_1 = 3150$, $f_2 = 3550$;

(c) $\frac{3}{32}$ in. × 3 in. plates at large spacing (Figure 6), $f_1 = 2400$, $f_2 = 2930$; and $f_1 = 1370$, $f_2 = 2170$.

There seems little doubt from the foregoing remarks that acoustic resonances are responsible for the high sound pressure levels observed and that, in the configurations tested, mechanical vibrations of the plates may be caused by the pressure fluctuations but play only a minor part in their generation.

### 4.2. FORCING MECHANISM

The close connection between the wake frequency of the single plates and the frequencies of the resonances observed with cascades of plates leaves little doubt that periodic wakes shed by the plates provide the energy input to the acoustic field.

The actual shedding of the vortices from each plate must be controlled by the ambient acoustic field in such a way that the input from all the plates is additive at certain discrete frequencies, which are related to the chord and spacing of the plates.

It is noticeable that the velocity range for each resonance is such that the wakes are shed at or below the natural wake frequency of the plates but never significantly above.

### 4.3. RELATION BETWEEN CASCADE GEOMETRY AND RESONANT FREQUENCIES

It was noted in relation to Forster's results that the frequencies calculated for standing waves in the annulus were some 7·5 to 9% above the observed values. If frequencies are calculated for the current tests using the spacing between nodes observed in the traversing plane the same result is obtained. In particular the large amplitude at 3200 c/s recorded in all configurations with plate spacings of 1·937 in. had one node at each plate. This spacing for plain uniform waves in space would correspond to a frequency of

$$\frac{1120}{2 \times 1{\cdot}937/12} = 3480 \text{ c/s},$$

i.e., a difference of 8·75%.

The acoustic mode at this frequency appears to be as shown in Figure 7 and a two-dimensional solution of the wave equation for this obtained by relaxation gives a frequency of 3300 c/s, so reducing the discrepancy between the calculated and observed values to 3%.

It is apparent that the prediction of resonant frequencies for any given configuration is not a simple matter and will probably require numerical solutions for each possible mode.

## 5. SUMMARY OF CONCLUSIONS

The results available from this initial investigation lead to the following conclusions.

(a) Resonances associated with the regular shedding of wakes from parallel plates in a fluid stream are, in some cases at least, caused by purely acoustic effects which are unrelated to the mechanical vibration of the plates.

(b) The amplitude of the pressure fluctuations can be of the order of $\pm \frac{1}{2}\rho v^2$ where $v$ is the fluid velocity.

(c) The frequencies are always at or a little below the natural wake shedding frequencies for isolated plates of the same dimensions.

(d) The similarity between the current results and Forster's observations shows that acoustic resonances can occur in both straight and annular unstaggered cascades of plates. The possibility of such resonances in staggered cascades such as turbine or compressor blades has still to be investigated.

(e) The frequency is generally somewhat lower than would be expected from the spacing of the nodes as observed upstream or downstream of the plates.

(*f*) Where the natural wake frequency corresponds to a wavelength considerably greater than twice the distance between plates the overall dimensions of the fluid passage control the frequency rather than the plate spacing.

(*g*) This type of resonance does not occur with round rods in place of plates.

## REFERENCES

1. V. T. FORSTER 1964–5 *Proc. Inst. Mech. Engrs.* **179**, 1–21. Communication on paper by W. RIZK and D. F. SEYMOUR. Investigations into the failure of gas circulators and circuit components at Hinkley Point Nuclear Power Station.

## LIST OF SYMBOLS

| | |
|---|---|
| $f$ | frequency in cycles per second |
| SPL | sound pressure level |
| $t$ | thickness |
| $v$ | velocity of fluid |
| $\rho$ | density of fluid (mass per unit volume) |
| $\omega$ | frequency in radians per second |
| $\lambda$ | sound wavelength |