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MONOPOLE RADIATION IN THE PRESENCE OF AN ABSORBING PLANE

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Two theoretical solutions for the field of a point sound source above a plane boundary between air and an absorbing material are discussed. The validity of approximations based on the plane-wave reflection coefficient and on the modified image method are also examined. Results of measurements made with a source in and above slabs of absorbent material are shown to be in agreement with theoretical results. The performance of free-field rooms having plane absorbent treatment and ground-reflection effects in the measurement of aircraft noise spectra can thus be predicted.

1. INTRODUCTION

The pressure-reflection coefficient for a plane sound wave travelling in a homogeneous isotropic medium of characteristic impedance Z_1 and incident at an angle θ_1 to the normal at a plane interface with a semi-infinite homogeneous isotropic medium of characteristic impedance Z_2 is

$$R_P = \frac{Z_2 \cos\theta_1 - Z_1 \cos\theta_2}{Z_2 \cos\theta_1 + Z_1 \cos\theta_2} \tag{1}$$

where θ_2 is the angle to the normal subtended by the wave refracted into the second medium, being defined by the relation

$$\gamma_2 \sin\theta_2 = \gamma_1 \sin\theta_1, \tag{2}$$

where γ_1 and γ_2 are the propagation coefficients in the two media.

For some applications simplified or approximate forms of equation (1) are used. For instance, to characterize the acoustical properties of a material, the normal incidence reflection coefficient ($\theta_1 = \theta_2 = 0$) may be used. On the other hand, it may be assumed that the material is "locally reacting", a term used to denote the conditions under which lateral propagation can be ignored; except for the special case of a material whose pores are all oriented perpendicular to the interface and thus "locally reacting" by definition, this requires that in the second medium either the attenuation coefficient is high, or that the propagation velocity is relatively low compared with that in the first medium. Again, the above expression for R_P, or a simplification thereof, is often used in situations where the incident wave is far from plane, simply because to do otherwise would involve a considerable increase in complexity. The reflection coefficient so derived permits qualitative analysis but may not yield quantitatively correct results.

However, an important problem arising in acoustics is the evaluation of the field of a spherically-radiating monopole source near the interface between air and a semi-infinite layer of absorbent material. Solutions for the electromagnetic field of a dipole situated above a finitely-conducting ground plane are well known [1, 2, 3] and solutions for the analogous acoustical case have also been given [4, 5]. However, available solutions are difficult to evaluate and the approximations necessary to effect a manageable solution are too restrictive for widespread use. The only practical applications made to date have been of the far-field approximation in attempts to account for observed frequency-dependent ground attenuation [6, 7, 8]. There are several important phenomena to which a full analysis might well have been applied but for the complexity of the near-field solutions. However, using a digital computer to perform certain integrations numerically, evaluation of the near field can be straightforward.

Following a brief review of the relevant analyses, this paper presents comparisons between theoretical and experimental results for the pressure distribution arising from a point sound source both in and near an absorbing plane surface.

2. SUMMARY OF THEORETICAL ANALYSES

Neglecting attenuation and non-linearity, sound propagation in air is described by the wave equation

$$(\nabla^2 + \gamma_1^2)\Phi = 0, \tag{3}$$

where Φ is the velocity potential, γ_1 is the propagation coefficient and a time-dependence $e^{-i\omega t}$ is assumed. The pressure p and particle-velocity $\mathbf{u}$ are given by

$$p = -\rho_0 \frac{\partial \Phi}{\partial t}$$

and

$$\mathbf{u} = \nabla \Phi,$$

where ρ_0 is the density of air. Solving in spherical polar co-ordinates, a point source thus produces spherical waves described by the relation

$$p = e^{i\gamma_1 r}/r \tag{4}$$

and the pressure amplitude falls off inversely with radial distance r. For a point source at a height h above an infinite rigid plane the solution for the pressure distribution can be obtained using the image concept. The field at all points above the plane is identical with that which would be produced by the original source plus that due to an image source of equal phase and amplitude symmetrically located at a distance h below the plane but with the plane itself removed. The justification for this treatment is that the necessary boundary conditions at the rigid plane are then fully satisfied. If r_1 is the distance between source and receiving point and r_2 is the distance between image and receiving point, the total pressure at a point is

$$p = e^{i\gamma_1 r_1}/r_1 + e^{i\gamma_1 r_2}/r_2. \tag{5}$$

However, this simplifying concept of a single point image is invalid when the surface is not perfectly reflecting, where it is not perfectly plane or where it is finite in extent, for in these cases no single point image or spherically symmetrical image strength will satisfy the required boundary conditions. Sometimes the assumption is made, without justification, that if the normal-incidence plane-wave pressure-reflection coefficient is R_n then the solution of the problem involving an absorbing plane can be approximated by using an image strength $|R_n|$ times the primary source strength. This method fails to give the correct solution, the error increasing as source and receiver approach the surface.

By their nature, spherical waves do not lend themselves to straightforward fitting of boundary conditions at a plane interface and, because the reflection coefficient for plane waves is well known, it is natural to proceed by expressing a spherical wave in terms of an expansion of plane waves. Considering an elementary plane wave with wavenormal $\boldsymbol{\gamma}_1$ from a point source at the origin, then for a point of observation $\mathbf{r}$ and using the integral representation

$$e^{i\gamma_1 r}/i\gamma_1 r = \int_{i\infty}^{1} e^{i\gamma_1 r\zeta}\,d\zeta, \tag{6}$$

it can be shown that

$$e^{i\gamma_1 r}/i\gamma_1 r = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi/2-i\infty} e^{i\boldsymbol{\gamma}_1\cdot\mathbf{r}}\sin\eta\,d\eta\,d\psi, \tag{7}$$

where η is the angle between $\boldsymbol{\gamma}_1$ and $\mathbf{r}$, and ψ is the equatorial angle. This integral in invariant with respect to rotation and it was rotation of the reference from the cartesian axis perpendicular to the surface to that of the radius vector $\mathbf{r}$ which enabled Weyl [2] to proceed with his solution of the electromagnetic problem. This integral representation, involving an infinite number of component plane waves and producing the desired singularity at the source point $\mathbf{r}=0$ but remaining finite elsewhere, includes inhomogeneous waves associated with complex angles.

Consider a source S at a height h above a portion of an infinitely large absorbing plane interface, as shown in Figure 1. If $R(\theta)=R(\psi,\eta)$ is the pressure-reflection coefficient for the component plane wave incident at an angle θ to the normal, then from equation (7) the total reflected field is

$$p_r = \frac{i\gamma_1}{2\pi}\int_0^{2\pi}\int_0^{\pi/2-i\infty} R(\psi,\eta)\,e^{i\gamma_1 r_2\cos\eta}\sin\eta\,d\eta\,d\psi, \tag{8}$$

where $\cos\theta=\cos\theta_0\cos\eta+\sin\theta_0\sin\eta\cos\psi$ and θ_0 is the angle between $\mathbf{r}_2$ and the outward normal to the surface. Equation (8) is evaluated by the method of steepest descents with $1-\cos\eta=is$, where s is a real variable, the major contribution arising from the region $s=0$ (i.e. $\eta=0$) which corresponds to geometric reflection. The effective image strength is

$$Q = \frac{\gamma_1 r_2}{2\pi}\int_0^{\infty} e^{-\gamma_1 r_2 s}\int_0^{2\pi} R(\psi,s)\,d\psi\,ds \tag{9}$$

while the total pressure at a point is given by

$$p = \frac{e^{i\gamma_1 r_1}}{r_1} + \frac{Q\,e^{i\gamma_1 r_2}}{r_2}. \tag{10}$$

Now Weyl obtained an asymptotic expansion for Q and hence solved for p by expanding the reflection coefficient in a Taylor series about the origin. Assuming the material is locally reacting, equation (1) reduces to

$$R(\theta) = \frac{\cos\theta - Y}{\cos\theta + Y}, \tag{11}$$

where Y is the admittance ratio of the two media, a complex quantity, and Ingard [5] showed that equation (9) then simplifies to give

$$Q = 1 - 2Y\gamma_1 r_2\int_0^{\infty}\frac{e^{-\gamma_1 r_2 s}\,ds}{\sqrt{(1+Y\Gamma_0+is)^2+(1-\Gamma_0^2)(1-Y^2)}}, \tag{12}$$

where $\Gamma_0 = \cos\theta_0$. The relative image strength, Q, is thus a function of the geometric reflection angle θ_0 and of the admittance ratio Y, and under certain conditions can assume a value greater than unity.

Having progressed thus far, Ingard considered only particular cases. For example, he dealt with the case when $Y=1$, which occurs when the impedance of the material matches that of air, and the case of normal incidence when $\Gamma_0 = 1$. Another approximation dealt with the case when $\gamma_1 r_2$ is very large, when the receiver is remote from the source, for in this case we can use the fact that the major contribution to the integral occurs for $s \ll 1$ and neglect the term in s^2 in the denominator. Under these conditions Q can be expressed in terms of the error function.

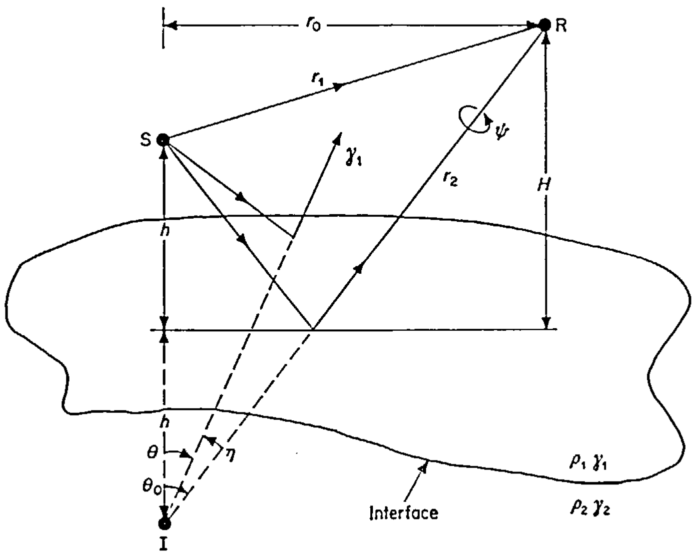

Figure 1. Geometry of source and receiver positions.

In the present work, equation (12) was integrated directly by numerical methods. The exponential in the numerator generally dominates both real and imaginary parts; even for complex Y the amplitude of oscillations about this exponential is relatively small. Simpson's rule provides a convenient integration procedure, an interval of 0·01 wavelength providing adequate accuracy for all practical purposes. Numerical data are presented in section 3 but meanwhile an alternative analysis is considered.

The form of solution of the wave equation (3) obviously depends on the co-ordinate system chosen. In cylindrical co-ordinates (r,ψ,z) the solution involves $J_0(\xi r)$ and $\exp(\pm z\sqrt{\xi^2-\gamma_1^2})$, and using the Fourier integral theorem it can be shown that for a single point source in free space, for $z>0$

$$\frac{e^{i\gamma_1 r}}{r} = \int_0^\infty \frac{J_0(\xi r)}{\sqrt{\xi^2-\gamma_1^2}} e^{-z\sqrt{\xi^2-\gamma^2}}\, \xi \mathrm{d}\xi, \tag{13}$$

provided that $i\gamma_1$ has a negative real part and the positive sign of the square root is taken to ensure the convergence of equation (13). This well-known result has several alternative derivations [9, 10], the field at any point (r,z) being represented as the sum of an infinite number of component waves of appropriate phase and amplitude. It has also been shown [11, 12, 19] that the same wave equations can describe propagation in porous absorbent materials having rigid framework provided the concept of complex density is introduced.

Thus, expressing both incident and reflected waves in a form analogous to equation (13) and fitting the boundary conditions at the interface between the two media to ensure continuity of pressure and normal component of particle velocity, the total potential is

$$\Phi = 2\rho_2 \int_0^\infty \frac{J_0(\xi r_0)}{\rho_2 l + \rho_1 m} e^{-(h+H)l} \xi \, d\xi, \tag{14}$$

where $l = (\xi^2 - \gamma_1^2)^{1/2}$ and $m = (\xi^2 - \gamma_2^2)^{1/2}$. This result was first given by Sommerfeld [1] for the electromagnetic case. Under certain restrictive conditions Sommerfeld was able to deform the contour of integration and to evaluate the contribution from the known singularities in terms of a power series. A further advance was made when Van der Pol [13], by introducing auxiliary variables, produced an interpretation in terms of the volume integral

$$\Phi = \frac{e^{i\gamma_1 r_1}}{r_1} + \frac{e^{i\gamma_1 r_2}}{r_2} - \frac{1}{\pi} \int\int\int \frac{\partial^2}{\partial z^2}\left(\frac{e^{i\gamma_2 r'}}{r'}\right) \frac{e^{i\gamma_1 r''}}{r''} d\tau, \tag{15}$$

where $d\tau = r\,dr\,dz\,d\phi$, a volume element,

$$r' = (r^2 + z^2)^{1/2},$$

$$r'' = \left[r_0^2 - 2rr_0 \cos\phi + r^2 + \left(h + H + \frac{\rho_2 z}{\rho_1}\right)^2\right]^{1/2},$$

r' being a real distance and r'' a complex distance. It should be noted that in this solution the general boundary conditions have been used and it has not been necessary to assume that the absorbing material is locally reacting. Equation (15) is exact under the conditions specified and represents a volume integral over the half-space below the usual image source position.

If now we assume that the lower medium is absorbing, so that γ_2 has a reasonably large imaginary part, the variation of r'' with r can be ignored and two of the three integrations can be performed directly, yielding

$$\Phi = \frac{e^{i\gamma_1 r_1}}{r_1} + \frac{e^{i\gamma_1 r_2}}{r_2} + 2i\gamma_2 \int_0^\infty \frac{e^{i\gamma_1 r''' + i\gamma_2 z}}{r'''} dz, \tag{16}$$

where $r''' = [r_0^2 + (h + H + \rho_2 z/\rho_1)^2]^{1/2}$, a complex quantity. Norton [3], having obtained this result for the electromagnetic case and Rudnick [4] in following through the analogous acoustical case, proceeded to approximate still further to obtain expressions valid only for large distances ($r_0 \gg h + H$). In fact, it was necessary to adjust the asymptotic expansion so produced to accord with an exact asymptotic expansion previously derived by Wise [14] direct from Sommerfeld's result, equation (14). The modification involves replacing Z_1/Z_2 in the far-field solution by

$$\frac{Z_1}{Z_2}\left(1 - \frac{\gamma_1^2}{\gamma_2^2} \sin^2\theta_0\right)^{1/2}. \tag{17}$$

As the discrepancy is due to the assumptions made when simplifying equation (15) it was further suggested by Rudnick and Norton that a similar correction term should be applied to equation (16). It will be seen from the results of section 3 that such a refinement makes only a slight difference to the calculated results and then only when both source and receiver are near the plane. In section 3, results calculated using this modified form for Z_1/Z_2 are labelled Rudnick-Norton.

The line integral of equation (16) is in a convenient form for immediate numerical evaluation giving results which remain valid for a wide range of separation of source and receiver. Such calculations have been carried out using a digital computer employing Simpson's rule with an integration increment of 0·02λ in most cases.

In fact, for the range of parameters employed in this work, evaluation shows that equations (16) and (12) yield results which agree to within 0·1 dB. This is to be expected for it is evident that the approximation made in deriving equation (16) from (15) is equivalent to assuming that the material is locally reacting. In circumstances where the locally reacting assumption is invalid it is necessary to revert to equation (15), but numerical evaluation then becomes more difficult.

Although other methods of solution [15] or variants of the above [10, 16, 17] are available they are not considered in detail here.

3. THEORETICAL AND EXPERIMENTAL DATA

There are few published data [18] on near-field pressure distributions due to a point source near an interface which have been measured with an absorbing material having specified acoustical characteristics. The following experimental work was therefore carried out.

The floor of a free-field (anechoic) room with internal dimensions 3·5 m long by 3·5 m wide by 2·5 m high was completely covered with slabs of absorbent material so as to present an unbroken plane surface. The material used was mineral wool (Stillite SR10) having a nominal bulk density of 160 kg m^{-3}; its mean specific flow resistance per unit length for air-flow normal to the surface was approximately $5{\cdot}5 \times 10^4$ MKS units and for air-flow in a direction parallel to the surface was about 30% less than this. The standard deviation about these mean values (including variation between slabs and within slabs) was of order 15%. It was concluded that although far from ideal in these respects, nevertheless the material was sufficiently homogeneous and isotropic to afford an experimental check on computed results. From measurements of characteristic impedance and propagation coefficient using a stationary-wave tube [19] it was established that the thickness of the material used, 0·1 m, was sufficient to ensure that the layer presented essentially its characteristic impedance to an incident wave down to a frequency of at least 0·5 kHz. The walls and ceiling of the free-field room were lined with highly absorbent wedges and at all frequencies used their amplitude-reflection coefficient was much lower than that of the plane material laid on the floor for these tests. In addition, source and receiver positions were kept well away from the wedged absorbent. Thus the presence of the walls of the room could be ignored and for all near-field measurements a reasonable approximation to a plane interface between air and a semi-infinite absorbent material was achieved.

The sound source consisted of a Reslo pressure unit type SU10 connected to a brass tube with internal diameter 3 mm and external diameter 6 mm and the open end of this was arranged to be in, or at various fixed distances above, the surface of the plane absorbent. The length of the source tube depended on the source height used but did not exceed 0·85 m. The field distribution was explored by making linear traverses perpendicular to the surface of the absorbent at various distances from the source, as shown in Figure 2, using a 0·8 m long probe tube with internal diameter 3 mm connected to a moving-coil microphone. The measured sound pressure level was recorded on a level recorder having direct mechanical linkage with the traversing mechanism, using either a 10 dB or 25 dB potentiometer.

Measurements were carried out at frequencies of 1, 2 and 4 kHz with fixed values of $h = 0$, 0·171, 0·342, 0·684 m whilst the microphone traversed the range $0 \leqslant H \leqslant 1{\cdot}25$ m. The precision achieved in measurements of distance was better than ±2 mm and in measurements of relative sound pressure level was of order ±0·1 dB.

Typical results for three different values of r_0 obtained with the source aperture flush with the plane of the absorbent are shown in Figure 3. Experimental results are shown by dotted

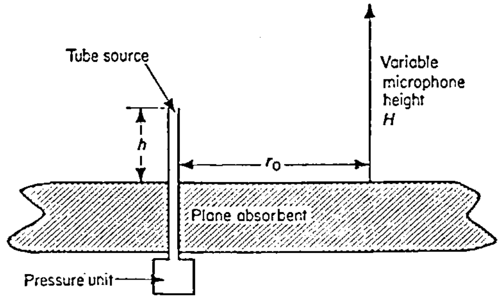

Figure 2. Experimental arrangement for exploration of sound field.

TABLE 1. *Mean acoustical characteristics of absorbing material*

	Z_2/Z_1	γ_2 (neper cm^{-1})
1 kHz	$2{\cdot}02+i1{\cdot}47$	$0{\cdot}44+i0{\cdot}33$
2 kHz	$1{\cdot}57+i0{\cdot}94$	$0{\cdot}69+i0{\cdot}47$

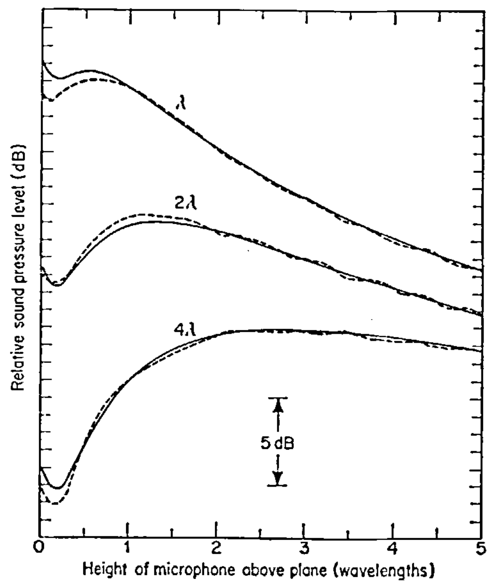

Figure 3. Sound field due to a point source in the surface of a plane absorbing material at 2 kHz. Parameter is r_0 expressed in wavelengths. ——, Calculated from Weyl-Ingard theory [equation (12)]. -----, Experimental result.

lines whilst the continuous lines show results computed from either equation (12) or (16) (which, as has been pointed out, give identical results) using values of impedance and propagation coefficient shown in Table 1. In Figure 3 (and also in Figure 6) the position on the ordinate scale is arbitrary and each curve merely shows the relative variation of sound pressure level with distance. In each case agreement between theory and experiment is excellent and confirms the evaluation method used to derive the theoretical data. For the same material at the same frequency the isobaric representation shown in Figure 4 clearly demonstrates the field distortion occurring in the neighbourhood of the absorbing surface. From Figure 5, which compares results calculated by four different methods, it is seen that

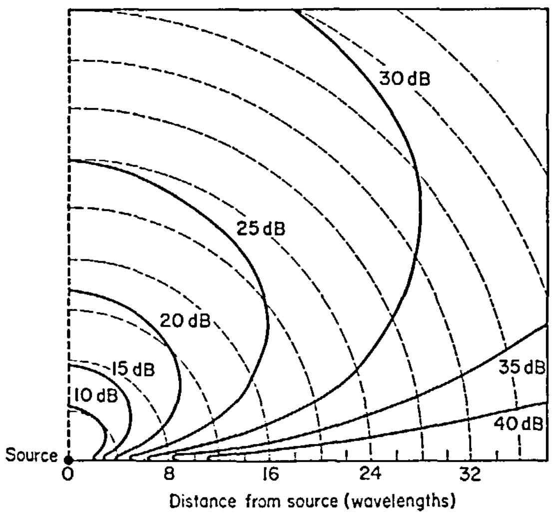

Figure 4. Pressure distribution round a point source in an absorbing plane (Stillite SR10, 2 kHz).

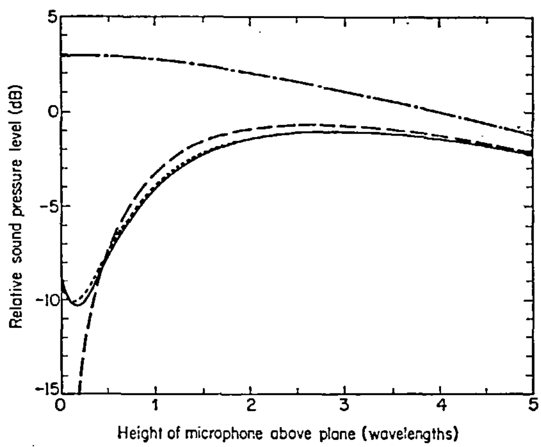

Figure 5. Calculated sound field due to a point source in the surface of a plane absorbing material at 2 kHz for $r_0 = 4\lambda$. ——, Weyl-Ingard theory [equation (12)]. -----, Rudnick-Norton theory [equations (16) and (17)]. — — —, Plane-wave reflection [equations (1) and (10)]. — · — ·, Simple image theory.

even in this case where the source is actually at the interface, the modified definition of r''' using (17) as proposed by Rudnick and Norton produces only a slight change in the computed field values. Except when the microphone is also very near the interface, use of the plane-wave reflection coefficient [equation (1)] with a modified image treatment [equation (10) with $Q = R_p$] provides a very good approximation for the field in the upper half-space. The simple image treatment using a constant image source strength is of little value in any region.

With the source above the plane, interference effects produce a more complicated field pattern, Figure 6 illustrating the excellent agreement obtained between theory and experiment for three source configurations. The typical variation of sound field with respect to angular rotation in the plane normal to the surface and passing through the source is shown in Figure 7 whilst Figure 8 compares results calculated by four different methods for the conditions specified for curve B of Figure 6. Again the image treatment gives poor results, especially near the plane, whilst the plane-wave approximation gives reasonably good agreement over the whole range.

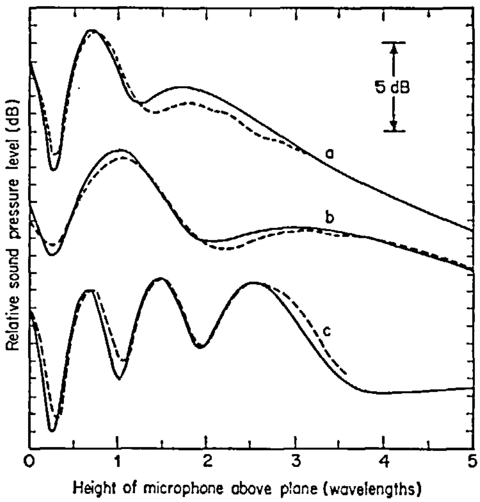

Figure 6. Sound field due to a point source above an absorbing plane. ——, Calculated from Weyl-Ingard theory [equation (12)]. ------, Experimental result. (See Table 2 for relevant parameters.)

TABLE 2 *Parameters relating to theoretical curves shown in Figure 6*

	Frequency, f (kHz)	Source height, h (wavelengths)	Separation, r_0 (wavelengths)
a	1	1	1
b	2	1	2
c	1	2	2

4. DISCUSSION

From the brief review given in section 2 and the results shown in section 3, it can be seen that similar results are obtained from the Weyl-Ingard result [equation (12)] or the approximate form of the Van der Pol result [equation (16)], both treatments requiring that the absorbing material be essentially locally reacting. Further, for the range of parameters used in the present study, the modification of equation (16) proposed by Rudnick and Norton to accord with Wise's asymptotic form for the far-field solution has only a slight effect on the calculated field. The method of solution whereby, for a given angle of incidence, the image source is ascribed a strength given by the plane-wave reflection coefficient [equation (1)] gives an excellent approximation over much of the upper half-space. Indeed, under most conditions and provided source or receiver are at least one half-wavelength from the plane, the error between results calculated by the more precise method and by this simplified method will be negligible for most practical purposes. In the far field, use of the plane-wave reflection coefficient can be justified [20] from the asymptotic solution of equation (4) but extension of its validity to near-field cases should find several important applications. The field

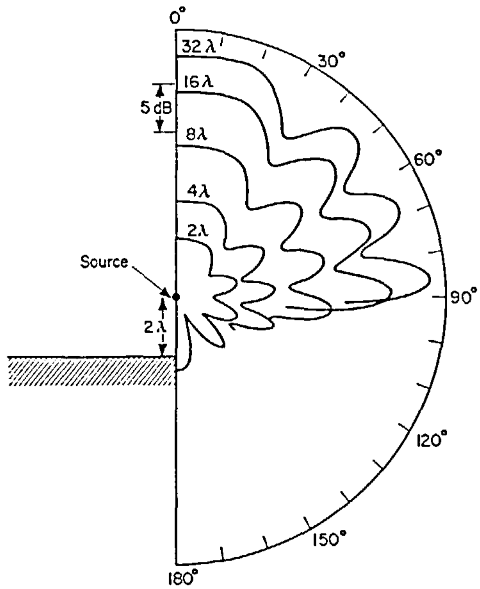

Figure 7. Angular variation of field round a point source 2λ above an absorbing plane (Stillite SR10, 2 kHz). Parameter is radial distance from source.

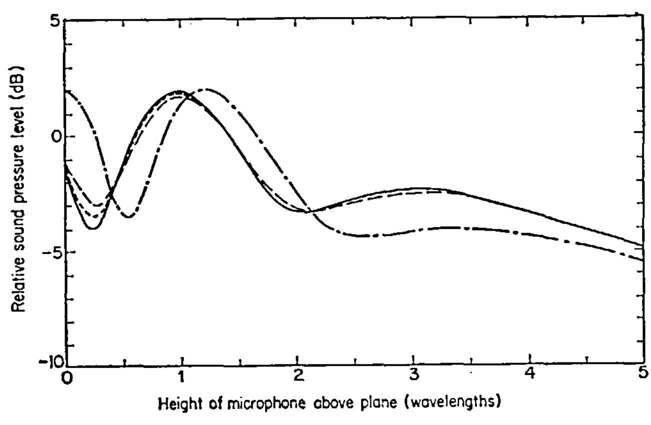

Figure 8. Calculated sound field due to a point source situated one wavelength above a plane absorbing material at 2 kHz for $r_0 = 2\lambda$. ——, Weyl-Ingard theory [equation (12)]. -----, Rudnick-Norton theory [equations (16) and (17)].———, Plane-wave reflection [equations (1) and (10)]. —·—·—, Simple image theory.

distribution inside an enclosure lined with plane absorbent material can be calculated [21], yielding valuable design data for test rooms used industrially for measuring the sound-power output of noise sources. Moreover, these results account for the effects of ground reflection in measurements of aircraft noise spectra [20] and may find practical application in predicting the high-frequency performance of absorbent-lined ducts.

Only under conditions where the lower medium has a very low attenuation constant and relatively high phase velocity is it necessary to employ the more exact solution [equation

(15)] and here again numerical solution is probably the simplest procedure. The maximum discrepancy between results calculated from equations (15) and (16) is to be expected when both the source and the receiver are near the surface of the absorbent ($h + H \to 0$).

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