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Research on vibration control of thin plate based on pre-stressing

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ABSTRACT

The weak dynamic stiffness of thin plate is one of the important factors that limit the use of thin plate. Improving the dynamic stiffness of thin plate is one of the effective methods for the vibration control of thin plate. In this paper, the influence of pre-stress on the vibration characteristics of thin plate is studied. A vibration control method of thin plate based on pre-stress is proposed. The vibration differential equation of quadrate thin plate under pre-stressing is established. Using the Galerkin principle, the natural frequencies corresponding to the shape functions of the quadrate thin plates under pre-stressing in different distribution forms are obtained. By comparison, it is found that pre-stressing on the thin plate can change the dynamic stiffness of thin plate. In particular, tensile stress can increase the dynamic stiffness of thin plate while compressive stress can reduce the dynamic stiffness of the thin plate. The greater the pre-stress, the more obvious the effect. In the end, the requirements of the pre-stress distribution which can improve the dynamic stiffness of thin plate effectively are derived.

1 INTRODUCTION

The application of lightweight products promotes the development of lighter and thinner mechanical structure, and thin plates are applied in more and more mechanical structures. The vibration control of thin plates is becoming one of the important research directions.

Vibration control includes vibration elimination, vibration isolation, vibration absorption, and structural dynamics modification and so on. According to the different control methods, vibration control can be divided into passive control, active control and semi-active control. With the development of computer technology, active control has been paid attention to by more and more

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scholars and companies[1-3]. However, structural dynamics modification, as the traditional vibration control method, is still widely used in design and manufacturing.

The core of structural dynamics modification is to improve the dynamic stiffness of the structure. The traditional structural dynamics modification is changing its mass and stiffness matrix by modifying the structure, so as to change the dynamic stiffness of the structure. With the research of relevant scholars, it's found that natural frequencies of structures are affected by pre-stresses[4] and a phenomenon also referred to as stress stiffening[5]. Chen et al.[6] studied the large amplitude vibration of an initially stressed cross ply laminated plates. They found that the frequency responses of nonlinear vibration were sensitive of the vibration amplitude, aspect ratio, thickness ratio, modulus ratio, stack sequence, layer number and state of initial stresses. Abd-Alla et al.[7] studied the vibration of an inhomogeneous hollow cylinder subjected to initial stress and magnetic field. The results showed that the inhomogeneity exponent, initial stress and magnetic field had an effect on the radial displacement, stresses and frequencies. Gao et al.[8] presented a method for calculating the natural frequency of rectangular thin plate with welding residual stress. Based on the calculation method, they obtained that the natural frequency was closely related to the magnitude of the residual stress, the order of the frequency, and the material. Yang et al.[9] studied the natural frequencies of underwater complex stress structures. A complex stress structure equation was established and solved for this type of structure.

The scholars above have studied the influence of pre-stress on the dynamic characteristics of thin plates, but the studies are not comprehensive enough. Based on the above research, this paper further studies the influence of different distribution forms of pre-stress on the vibration characteristics of rectangular thin plate, and proposes to improve the dynamic stiffness of the thin plate by means of active pre-stressing, so as to realize vibration control of the thin plate through pre-stressing.

2 MODELING AND SOLUTION OF VIBRATION DIFFERENTIAL EQUATIONS FOR SIMPLY SUPPORTED PRE-STRESSED RECTANGULAR THIN PLATES

The behavior of isotropic plates is well known and has been treated in detail by several authors[10–12].

In this section, based on the theoretical research, the rectangular thin plate shown in Fig.1 is taken as the research object. With the simple support of the four sides as the constraint form, the vibration differential equations of simple supported pre-stressed rectangular thin plates are established and solved.

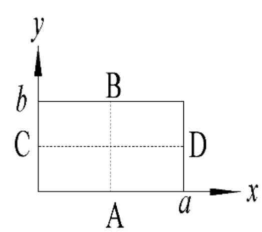


Fig. 1 – Quadrate thin plate.

As is known to all, the differential equation of forced, undamped motion of plates has the form[12]

$$D\nabla^2\nabla^2 w(x,y,t)+\rho h\frac{\partial^2 w}{\partial t^2}(x,y,t)=p(x,y,t) \tag{1}$$

Where

$$D = \frac{Eh^3}{12(1-\mu^2)} \tag{2}$$

is the flexural rigidity of the plate. E is the elastic modulus, h is the thickness of thin plate, μ is the Poisson's ratio, ρ is the mass density, $w(x, y, t)$ is the deflection of thin plate, $p(x, y, t)$ is the force, $\nabla^2()$ is the Laplace operator, given by

$$\nabla^2(w) = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \tag{3}$$

The force component of the pre-stress in the vibration direction of thin plate is analyzed when pre-stress exists in the thin plate. The thin plate bends when it vibrates. As can be seen from Fig.2, when the thin plate bends, the pre-stress in the x direction produces a component force in the vibration direction (z direction) of the thin plate. Assuming that the tensile stress is positive for the thin plate, the component of the pre-stress in the x direction of the unit thickness on the dxdy face is[13].

$$f_{xz} = \sigma_x \mathrm{d}y\left(\theta + \frac{\partial \theta}{\partial x}\mathrm{d}x\right) - \sigma_x \mathrm{d}y\theta = \sigma_x \frac{\partial^2 w}{\partial x^2}\mathrm{d}x\mathrm{d}y \tag{4}$$

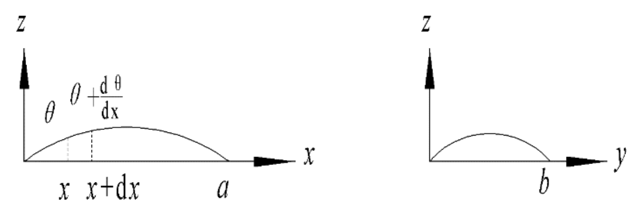


Fig. 2 – Deformation schematic diagram of the of thin plate.

The analysis method of the pre-stress in the y direction is the same as the above method. Therefore, the total component of the x, y direction of thin plate in the z direction is

$$f_z = f_{xz} + f_{yz} = \sigma_x \frac{\partial^2 w}{\partial x^2}\mathrm{d}x\mathrm{d}y + \sigma_y \frac{\partial^2 w}{\partial y^2}\mathrm{d}x\mathrm{d}y \tag{5}$$

Therefore, the force p per unit area in the z direction caused by the pre-stress is[13]

$$p = \int_{-h/2}^{h/2}\left(\sigma_x \frac{\partial^2 w}{\partial x^2} + \sigma_y \frac{\partial^2 w}{\partial y^2}\right)\mathrm{d}z = h\left(\sigma_x \frac{\partial^2 w}{\partial x^2} + \sigma_y \frac{\partial^2 w}{\partial y^2}\right) \tag{6}$$

When Eqn. (6) is taken into Eqn. (1), the vibration differential equation of the pre-stressed thin plate can be obtained, which is shown as Eqn. (7).

$$D\nabla^2\nabla^2 w(x, y, t) + \rho h \frac{\partial^2 w}{\partial t^2}(x, y, t) = h\left(\sigma_x \frac{\partial^2 w}{\partial x^2} + \sigma_y \frac{\partial^2 w}{\partial y^2}\right) \tag{7}$$

For the rectangular thin plate, when the constraint mode with the simple support of four sides is adopted, the displacement satisfying the boundary condition is[14-15]

$$w(x, y, t) = T(t)\cdot W(x, y) \tag{8}$$

Where $W(x, y)$ can be set

$$W(x, y) = \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{9}$$

According to the Galerkin principle, $w(x, y, t)$ satisfies

$$\iint_s \left(D\nabla^4 w(x, y, t) + \rho h\ddot{w} - q\right)W\mathrm{d}s = \iint_s \left[D\nabla^4 w(x, y, t) + \rho h\ddot{w} - h\left(\sigma_x \frac{\partial^2 w}{\partial x^2} + \sigma_y \frac{\partial^2 w}{\partial y^2}\right)\right]W\mathrm{d}s = 0 \tag{10}$$

Eqn. (10) can be converted to

$$\iint_s \left[D\nabla^4 W - h\left(\sigma_x \frac{\partial^2 W}{\partial x^2} + \sigma_y \frac{\partial^2 W}{\partial y^2} \right) \right] \cdot W \mathrm{d}s \cdot T + \iint_s (\rho h W \cdot W) \mathrm{d}s \cdot \ddot{T} = 0 \tag{11}$$

Therefore, the natural frequency corresponding to each order shape function of the rectangular thin plate can be obtained as

$$\omega_{mn}{}^2 = \frac{\iint_s \left[D\nabla^4 W - h\left(\sigma_x \frac{\partial^2 W}{\partial x^2} + \sigma_y \frac{\partial^2 W}{\partial y^2} \right) \right] \cdot W \mathrm{d}s}{\iint_s (\rho h W \cdot W) \mathrm{d}s} \tag{12}$$

3 RESEARCH ON VIBRATION CHARACTERISTICS OF SIMPLY SUPPORTED RECTANGULAR THIN PLATE UNDER UNIDIRECTIONAL UNIFORM PRE-STRESS

The vibration characteristics generally include the natural frequency, vibration shape, etc. This paper only considers the pre-deformation in the elastic range. At this time, the vibration shape of thin plate usually remains unchanged. Therefore, only the natural frequency is analyzed in this paper.

3.1 Analysis of Natural Frequencies of Rectangular Thin Plate under Unidirectional Uniform Pre-Stress

According to the analysis in above section, the natural frequency of thin plate under each stress distribution can be calculated according to Eqn. (12). Without loss of generality, the vibration characteristics of thin plates under unidirectional uniform pre-stress are analyzed, which is taken as an example. It's assumed that there is uniform pre-stress in the x direction of the thin plate and no pre-stress in the y direction, i.e.

$$\begin{cases} \sigma_x = \sigma_0 (\text{constant}) \\ \sigma_y = 0 \end{cases} \tag{13}$$

Take m=n=1, i.e. analyze the first-order natural frequency, and substitute Eqn. (13) into Eqn. (12). The first-order natural frequency can be derived as Eqn. (14).

$$\omega_{11}{}^2 = \frac{\iint_s \left[D\nabla^4 \left(\sin\frac{\pi x}{a} \sin\frac{\pi y}{b} \right) - h\sigma_0 \frac{\partial^2}{\partial x^2} \left(\sin\frac{\pi x}{a} \sin\frac{\pi y}{b} \right) \right] \cdot \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} \mathrm{d}s}{\iint_s \left(\rho h \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} \cdot \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} \right) \mathrm{d}s} = \frac{D}{\rho h} \left[\left(\frac{\pi}{a} \right)^2 + \left(\frac{\pi}{b} \right)^2 \right]^2 + \frac{\pi^2 \sigma_0}{\rho a^2} \tag{14}$$

According to elastic mechanics, the first-order natural frequency of a rectangular thin plate with four edges simply supported without pre-stress is

$$\omega_0{}^2 = \frac{D}{\rho h} \left[\left(\frac{\pi}{a} \right)^2 + \left(\frac{\pi}{b} \right)^2 \right]^2 \tag{15}$$

By the comparison of Eqn. (14) and (15), it can be found that when the tensile stress exists in the thin plate, the natural frequency increases; when the compressive stress exists in the thin plate, the natural frequency decreases. In addition, the greater the pre-stress of the thin plate, the greater the effect on the natural frequency of the thin plate.

Actually, the analysis method of high-order natural frequency is the same as that of the first natural frequency. The detailed calculation for high-order natural frequency is not described in the paper and its expression is shown as Eqn. (16). The coupling between the x and y directions is ignored in the calculation process.

$$\omega_{mn}{}^2 = \frac{D}{\rho h}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^2 + \frac{1}{\rho}\left[\sigma_{x0}\cdot\left(\frac{m\pi}{a}\right)^2 + \sigma_{y0}\cdot\left(\frac{n\pi}{b}\right)^2\right] \tag{16}$$

3.2 Finite Element Verification

In order to verify the correctness of Eqn. (12), ABAQUS is used as the analysis software to perform modal analysis on the pre-stressed simply supported thin plate and an Eigen-vector analysis is carried out. The dimensions are as follows: length $a = 1m$, width $b = 1m$, thickness $h = 0.004m$, density $\rho = 7700 kg/m^3$, modulus of elasticity $E = 2e11Pa$, Poisson's ratio $\mu = 0.3$. In order to pre-stress on the thin plate, the following constraints are used: edge C is simply supported; edges A and B constrain y and z degrees of freedom; edge D constrains y and z degrees of freedom, and the displacement L_x in the x direction, as shown in Fig.3.

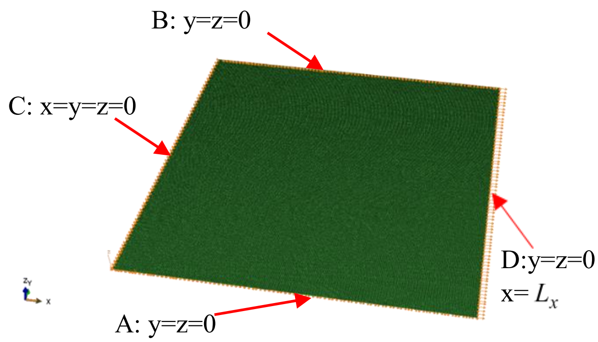
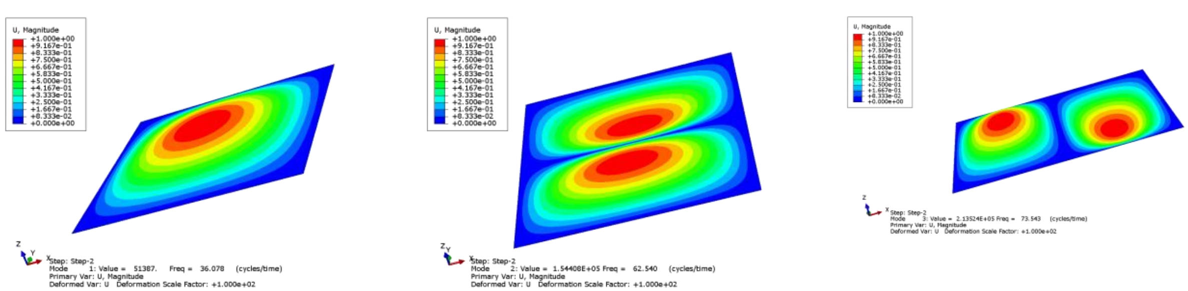


Fig. 3 – Constraints of thin plate.

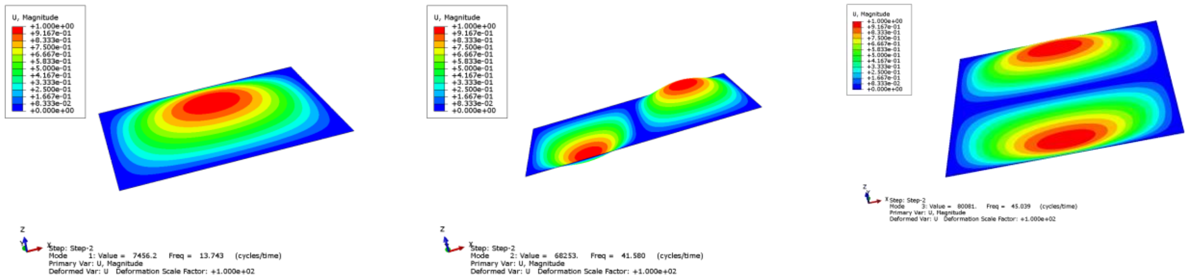
Through the finite element analysis, the calculation results shown in Table 1 can be obtained. The first three vibration shapes are shown in Fig.4.

Table 1 - Comparison between analytical method and finite element method

Order	Natural frequency/Hz				
	Stress-free	Stress（$L_x = 0.1mm$）$\sigma_x = 22MPa, \sigma_y = 6.6MPa$		Stress（$L_x = -0.02mm$）$\sigma_x = -4.4MPa, \sigma_y = -1.3MPa$	
		Analytical method	Finite element method	Analytical method	Finite element method
1	19.38	36.11	36.08	13.80	13.74
2	48.45	62.60	62.54	41.64	41.58
3	48.45	73.61	73.54	45.12	45.04
4	77.52	98.61	98.47	72.59	72.43



First order (tensile stress) *Second order (tensile stress)* *Third order (tensile stress)*



First order (compressive stress)） *Second order (compressive stress)* *Third-order (compressive stress)*

Fig. 4 – Vibration shapes of thin plate under tensile/compressive stress.

The results shown in Table 1 and Fig.4 verify the above conclusion that the natural frequency of thin plate increases when there is tensile stress in the thin plate; when the compressive stress exists in the thin plate, the natural frequency decreases. At the same time, by the comparison of the vibration shapes, the vibration shape of the thin plate remains unchanged while the natural frequency changes. It also verifies that the mutual coupling in the x and y directions can be neglected.

3.3 The Influence of the Pre-stress on the Natural Frequency of the Rectangular Thin Plate

Based on the above section, the influence of the stress distribution region on the natural frequency of rectangular thin plate is further studied. In order to simplify the analysis, the effects of the stress distribution of Eqn. (13) are still analyzed as an example. The thin plate is equally divided into four parts in the x-direction, as shown in Fig.5. The pre-stress of the thin plate is distributed according to Table 2, and the effect of the stress distribution region on the vibration characteristics of the thin plate is analyzed where the generation method of pre-stress is ignored. According to the symmetry, the analysis of effect of the pre-stress in the y direction is the same as that in the x direction.

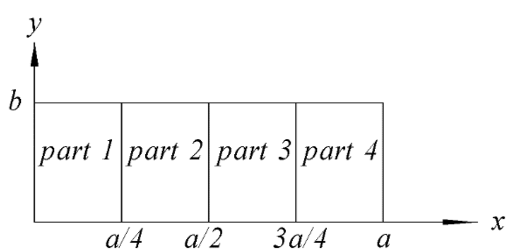


Fig. 5 – Block diagram of thin plate.

Table 2 - Pre-stress distribution of rectangular thin plate

Group	Stress			
	Part 1	Part 2	Part 3	Part 4
1	1	1	1	1
2	1	1	-1	-1
3	-1	-1	1	1
4	1	0	0	-1
5	0	1	-1	0
6	1	0	-1	0
7	0	1	0	-1

Where "1" represents the tensile stress whose magnitude is σ_0, "0" represents no pre-stress, "-1" represents the compressive stress whose magnitude is σ_0.

Generally, the first-order natural frequency is the most concerned frequency. Therefore, the first-order natural frequency of the thin-plate is analyzed emphatically in this paper.

According to Eqn. (12), the first-order natural frequencies of different distributions shown in Table 3 can be obtained and their increments are plotted as shown in Fig.6.

Table 3 - First-order natural frequencies of different distributions

Group	ω_{11}^2	Increment relative to ω_0^2
1	$\frac{D}{\rho h}\left[\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2\right]^2+\frac{\pi^2\sigma_0}{\rho a^2}$	$\frac{\pi^2\sigma_0}{\rho a^2}$
2	$\frac{D}{\rho h}\left[\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2\right]^2$	0
3	$\frac{D}{\rho h}\left[\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2\right]^2$	0
4	$\frac{D}{\rho h}\left[\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2\right]^2$	0
5	$\frac{D}{\rho h}\left[\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2\right]^2$	0
6	$\frac{D}{\rho h}\left[\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2\right]^2-\frac{\pi\sigma_0}{\rho a^2}$	$-\frac{\pi\sigma_0}{\rho a^2}$
7	$\frac{D}{\rho h}\left[\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2\right]^2+\frac{\pi\sigma_0}{\rho a^2}$	$\frac{\pi\sigma_0}{\rho a^2}$

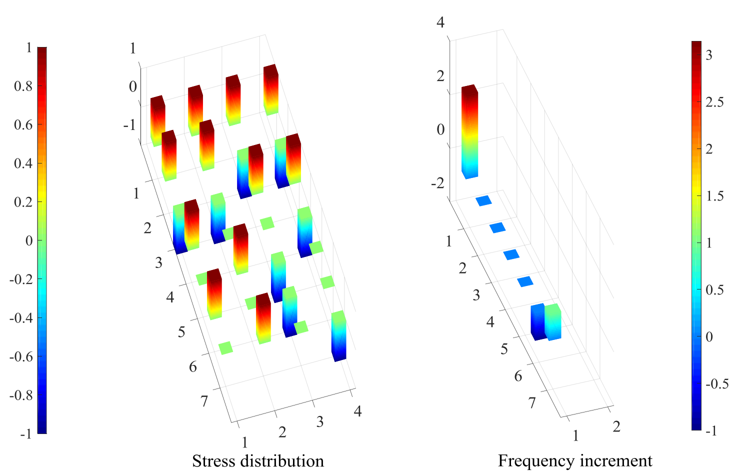


Fig. 6 – Three-dimensional histogram of natural frequency increment.

According to the results shown in Table 3 and Figure 6, the following conclusions can be drawn.

(1) From the 1,2,3,4,5 groups of pre-stress distributions, it can be found that the influence of pre-stress on the first-order natural frequency of rectangular thin plate is symmetrical with respect to the centerline of thin plate (AB or CD).

(2) From the 1, 6 and 7 groups of pre-stress distributions, it can be found that the closer the pre-stress to the center line (AB or CD) of the thin plate, the greater the influence of the pre-stress on the first-order natural frequency of thin plate.

4 VIBRATION CHARACTERISTICS ANALYSIS OF SIMPLY SUPPORTED RECTANGULAR THIN PLATE UNDER HARMONIC DISTRIBUTION PRE-STRESS

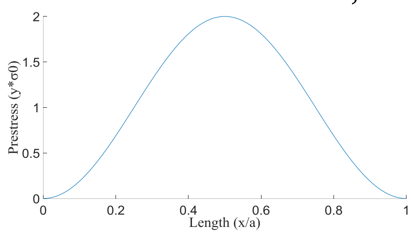
Uniform pre-stress is the simplest form of pre-stress distribution. Actually, there are a variety of pre-stress distributions through different installation and processing methods. For example, welding can generate harmonic distribution pre-stress[8,13]. According to the Fourier series, each pre-stress can be decomposed into multiple harmonic functions with the periodical extension method when the sufficient conditions of Dirichlet are satisfied. Therefore, the analysis of the natural frequencies of simply supported rectangular thin plates under harmonic distribution pre-stress is the basis for that of thin plates under the general pre-stress.

In order to compare with the uniform pre-stress, the two unidirectional harmonic distributions shown in Fig.7 are selected, i.e. Eqn. (17), (18).

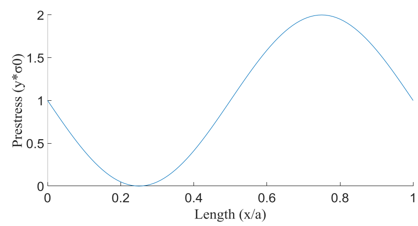
$$\begin{cases} \sigma_x = -\sigma_0 \cdot \cos\dfrac{2\pi x}{a} + \sigma_0 \\ \sigma_y = 0 \end{cases} \quad \text{（Cosine function）} \tag{17}$$

$$\begin{cases} \sigma_x = -\sigma_0 \cdot \sin\dfrac{2\pi x}{a} + \sigma_0 \\ \sigma_y = 0 \end{cases} \quad \text{（Sine function）} \tag{18}$$

Eqn. (17) and (18) are taken into Eqn. (12) to calculate the first-order natural frequencies and increments of the two distributions, as shown in Table 4.



(a) Cosine function



(b) Sine function

Fig. 7 – Harmonic distribution pre-stress.

Table 4 - First-order natural frequency of the harmonic distributions

Group	ω_{11har}^{2}	Increment relative to ω_0^{2}	Increment relative to ω_{11}^{2}
1（Cosine）	$\dfrac{D}{\rho h}\left[\left(\dfrac{\pi}{a}\right)^2+\left(\dfrac{\pi}{b}\right)^2\right]^2+\dfrac{3\pi^2\sigma_0}{2\rho a^2}$	$\dfrac{3\pi^2\sigma_0}{2\rho a^2}$	$\dfrac{\pi^2\sigma_0}{2\rho a^2}$
2（Sine）	$\dfrac{D}{\rho h}\left[\left(\dfrac{\pi}{a}\right)^2+\left(\dfrac{\pi}{b}\right)^2\right]^2+\dfrac{\pi^2\sigma_0}{\rho a^2}$	$\dfrac{\pi^2\sigma_0}{\rho a^2}$	0

The stress distributions in Eqn. (17) and (18) satisfy

$$\int_0^a \sigma_x \mathrm{d}x = \sigma_0 \tag{19}$$

According to the results in Table 4, the first-order natural frequency of sine distribution is equal to that of uniform distribution while $\sigma_x(x)-\sigma_0 = -[\sigma_x(a-x)-\sigma_0](0 \le x \le a/2)$ in sine function. Therefore, conclusion (1) in above section is further verified. Similarly, the first-order natural frequency of cosine distribution is high than that of uniform distribution while $\sigma_x'(x) \ge 0$ and $\sigma_x(x) = \sigma_x(a-x) \quad (0 \le x \le a/2)$ in cosine function. Therefore, conclusion (2) in above section is further verified. Moreover, in combination with Eqn. (19), it can be seen that the influence of the pre-stress on the first-order natural frequency of the rectangular thin plate in the form of the cosine distribution described in Eqn. (17) is greater than that of the uniform pre-stress on the natural frequency.

In fact, the method to increase the natural frequencies of thin plates by means of pre-stressing requires not only the analysis of Eqn. (12) but also the ease of installation and processing methods as considerations. Therefore, this paper only presents the requirements for the pre-stress distribution of improving the dynamic stiffness of thin plate.

(1) Pre-stress should be tensile stress while compressive stress should be avoided as much as possible.

(2) If conditions allow, the pre-stress should be as large as possible (satisfying the elastic deformation range).

(3) The distribution of pre-stress should concentrate on the centerline of thin plate (AB or CD).

The requirement (3) is mainly used to increase the first-order natural frequency.

Therefore, the harmonic distribution pre-stress is an appropriate form of pre-stress distribution.

5 CONCLUSIONS

In this paper, the rectangular thin plate is taken as the research object. Improving the dynamic stiffness of thin plate is taken as the method. The vibration control of thin plate is taken as the purpose. The method of controlling the vibration of thin plate by pre-stressing is proposed. Galerkin principle is used in this paper to obtain the natural frequency expression of rectangular thin plate under pre-stress. The effect of the distribution of unidirectional uniform pre-stress and harmonic distribution pre-stress on the thin plate is analyzed based on the expression and the following conclusions are drawn which provide guidancee for vibration control with pre-stress.

(1) Tensile pre-stress can increase the natural frequency of thin plate, while the compressive pre-stress can reduce the natural frequency of thin plate instead.

(2) The greater the tensile pre-stress, the higher the natural frequency of thin plate.

(3) The closer the pre-stress to the center line (AB or CD) of thin plate, the greater the influence of the pre-stress on the first-order natural frequency of thin plate.

The relationship between the loading way and the pre-stressing is not considered. However, the relationship is also one of the key steps to improve the dynamic stiffness of the thin plate to achieve the vibration control of the thin plate by pre-stressing. Therefore, the further research is needed.

6 ACKNOWLEDGEMENTS

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