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FREE VIBRATION ANALYSIS OF TWO-STAGE PLANETARY GEAR WITH FRICTION

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ABSTRACT

Considering the effect of teeth surface sliding friction, free vibration of two-stage planetary gears (TPG) is studied theoretically for the first time. The lateral-torsional coupling dynamic model and equation are established with three degrees of freedom: two translations and one rotation. The change rule of natural frequency is discussed with the case of first stage planetary gear's number 4 and second stage planetary gear's number 3, 4 and 5. Afterwards three vibration modes are summarized by calculating the free vibration. In order to understand the behavior of friction, the effect of friction on natural frequencies is analyzed for the case of considering friction and not considering friction. Furthermore, the 'self-coupling' phenomenon is obtained from the vibration of center component of TPG. Meanwhile, the 'mutual coupling' is obtained between the first-stage planetary gear (FPG) and the second-stage planetary gear (SPG).

INTRODUCTION

Due to the advantage of large transmission ratio and strong carrying capacity, planetary gears are widely used in ship shafting device, wind power transmission, aerospace and other fields. However, the problems of vibration and noise are particularly serious. According to reliable reports, the noise in the helicopter can exceed 100 dB caused by the vibration of planetary gear. Thus, it is of great practical significance to study the vibration characteristics of planetary gear.

At present, many scholars have studied the free vibration of planetary gear. As early as 1994, Kahraman has simplified the planetary gear model, and studied the free vibration for the case of only considering torsional degree of freedom for the first time [1]. Based on the research of Kahraman, Parker constructed the lateral-torsional coupling dynamics model of planetary gear. He classified the vibration modes into three types: rotational, translational and planet modes. This research provided a modeling calculation method of planetary gears [2-3]. Dhouib proposed a compound planetary gear train to study the free vibration. His research indicates that the change of planets' angular position does not affect vibration mode [4]. Kahraman developed a family of torsional dynamic models of compound gear sets to predict the free vibration characteristics under different kinematic configurations resulting in different speed ratios [5]. Qian proposed a lateral-torsional coupled dynamic

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model, derived the associated reduced-order eigenvalue for each type of vibration mode and presented the analytical expression of natural frequency for planet mode [6]. By designing a two-stage closed-form planetary gear, Zhang obtained the natural frequencies and vibration modes. Meanwhile, the finite element model is established to validate the lumped-parameter model [7]. Huang developed a purely torsional dynamic model of closed-form planetary gear set to investigate its natural frequency and free vibration modes [8]. Sheng investigated the vibration modal properties of double-helical planetary gear system and he categorized the vibration modes into three essentially different types of modes including planet mode, rotational-axial mode, and planer-translational mode [9]. Eritenel studied the modal properties of three dimensional helical planetary gears [10]. By simplifying the compound planetary gears, only considering the rotational degree, Guo studied the natural frequencies and vibration modes. However, the purely torsional model is simple, and the lateral-torsional coupling model need further be considered [11]. Parker analyzed the vibration modes of planetary gears with unequally spaced planets and an elastic ring gear [12]. Tristan established the two spur planetary gears models and measured the rotational and translational vibration using experiment. Their researches verified that the lumped-parameter and finite element models could be used effectively to predict the natural frequencies and modal properties established by experimentation [13]. From the theoretical and experimental point of view, the above scholars have made many research on the free vibration of planetary gears, and obtained some meaningful conclusions, which laid a theoretical foundation for the study of planetary gear.

At present, the friction's effect on the dynamic characteristics of planetary gears has also been considered by scholars. Vaishya analyzed the non-linear and parametric effects caused by the teeth surface sliding friction [14]. Velex presented an analysis of teeth surface friction excitations. And the effects of teeth surface friction, time varying meshing stiffness and other parameters on the dynamic characteristics are also discussed [15]. Liu studied the influences of friction on parametric instabilities and dynamic response of a single-mesh gear pair [16]. He considered the influences of sliding friction on the dynamics of spur gear pair with time-varying stiffness, and studied the sliding friction' effects for the helical gears [17-18]. Csoban studied the effect of teeth friction losses on the efficiency of planetary gears [19]. The above literatures considering friction mainly focus on vibration response. However, few reports of free vibration of planetary gears with friction can be found. In this paper, the TPG dynamic model with friction is studied for the first time. The effects of friction on natural frequency and vibration modes are discussed for the case of considering friction and not considering friction. This paper builds a theoretical foundation for the research of vibration response.

This paper is consisted of five parts. In Section I, the TPG dynamics model is established. The force analysis is done between the sun and planets, planets and ring, planets and carrier. Also, the dynamic equation of component of planetary gear is derived. In Section II, free vibration of the TPG is calculated. Natural frequencies obtained by considering friction are compared with those not considering friction. The mechanism of friction on natural frequency is also revealed. In Section III, the 'self-coupling' phenomenon and 'mutual coupling' phenomenon are obtained from the vibration of center component of TPG. Section IV is Conclusion. Section V is Acknowledgements.

DYNAMIC MODELING OF PLANETARY GEAR

Consider a TPG model shown in Figure 1 and Figure 2. The whole system consists of the FPG containing carrier Z_n^{I}, ring Z_r^{I}, sun Z_s^{I}, planets Z_p^{I} and the SPG containing carrier Z_n^{II}, ring Z_r^{II}, sun Z_s^{II}, planets Z_p^{II}. The ring of the FPG and SPG are fixed. The power is input from the sun in the first-stage, and output from carrier in the second-stage. T_s^{I} is the loading torque applied to the sun. T_s^{II} is the output torque of carrier.

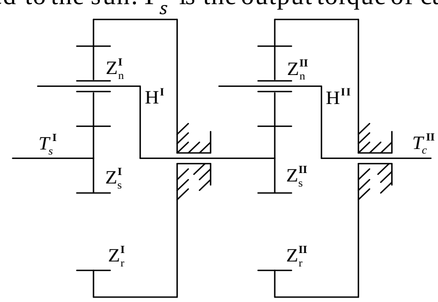

Figure 1. Dynamic model of TPG.

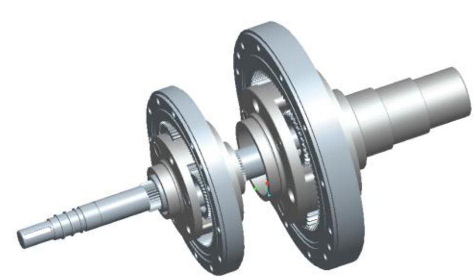

Figure 2. Geometric model of TPG.

The lumped parameter model with three degrees of freedom for the lateral-torsional coupling dynamic model can be obtained by Lin and Parker [2]. Different from Lin and Parker, in this paper, the relative position of center part of planetary gear is given, as shown in Figure 3. Here, Figure 3a gives the position relation of sun and planet. Figure 3b gives the position relation of ring and planet. Figure 3c gives the position relation of carrier and planet.

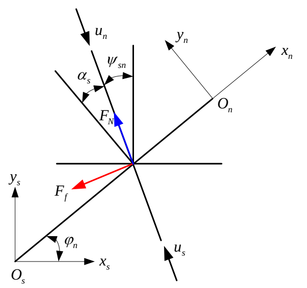

Figure 3a. Sun-Planet meshing.

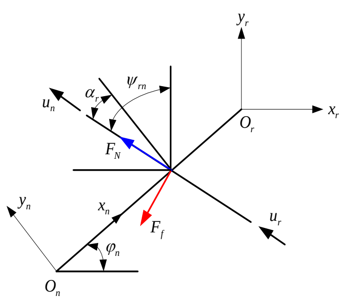

Figure 3b. Ring-Planet meshing.

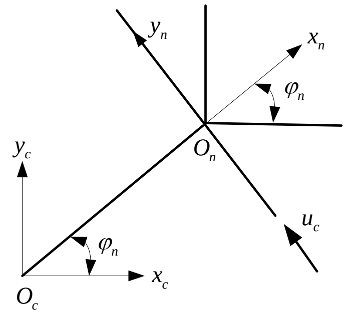

Figure 3c. Carrier-Planet meshing.

The internal force of gear is mainly composed of three parts: exciting force generated by meshing stiffness, static transmission error, sliding friction. mj, Ij/r_j^2 (j=c,r,s,p) is the mass and equivalent moment of inertia for each component. After making a detail analysis for the components of planetary gear, the specific dynamic equation with friction is given as follows.

1) Figure 3a depicts that the sun is subjected to the meshing force from the planets and the supporting. Equations of sun of the FPG are

$$\begin{cases} m_s^{\mathrm{I}}(\ddot{x}_s^{\mathrm{I}} - 2\omega_c \dot{y}_s^{\mathrm{I}} - \omega_c^2 x_s^{\mathrm{I}}) - (\sin\psi_{sn} + \lambda\mu_s^{\mathrm{I}}\cos\psi_{sn})\sum_{n=1}^{N}[k_{sp}^{\mathrm{I}} f(\delta_{sn}^{\mathrm{I}}) + c_{sp}^{\mathrm{I}}\dot{\delta}_{sn}^{\mathrm{I}}] + k_{sx}^{\mathrm{I}} x_s^{\mathrm{I}} + c_{sx}^{\mathrm{I}}\dot{x}_s^{\mathrm{I}} = 0 \\ m_s^{\mathrm{I}}(\ddot{y}_s^{\mathrm{I}} + 2\omega_c \dot{x}_s^{\mathrm{I}} - \omega_c^2 y_s^{\mathrm{I}}) + (\cos\psi_{sn} - \lambda\mu_s^{\mathrm{I}}\sin\psi_{sn})\sum_{n=1}^{N}[k_{sp}^{\mathrm{I}} f(\delta_{sn}^{\mathrm{I}}) + c_{sp}^{\mathrm{I}}\dot{\delta}_{sn}^{\mathrm{I}}] + k_{sy}^{\mathrm{I}} y_s^{\mathrm{I}} + c_{sy}^{\mathrm{I}}\dot{y}_s^{\mathrm{I}} = 0 \\ \dfrac{I_s^{\mathrm{I}}}{(r_s^{\mathrm{I}})^2}\ddot{u}_s^{\mathrm{I}} + (1 - \lambda\mu_s^{\mathrm{I}}\dfrac{l_{sn}^{\mathrm{I}}}{r_s^{\mathrm{I}}})\sum_{n=1}^{N}[k_{sp}^{\mathrm{I}} f(\delta_{sn}^{\mathrm{I}}) + c_{sp}^{\mathrm{I}}\dot{\delta}_{sn}^{\mathrm{I}}] + k_{su}^{\mathrm{I}} u_s^{\mathrm{I}} + c_{su}^{\mathrm{I}}\dot{u}_s^{\mathrm{I}} = \dfrac{T_s^{\mathrm{I}}}{r_s^{\mathrm{I}}} \end{cases} \tag{1}$$

Equations of sun of the SPG are

$$\begin{cases} m_s^{\mathrm{II}}(\ddot{x}_s^{\mathrm{II}} - 2\omega_c \dot{y}_s^{\mathrm{II}} - \omega_c^2 x_s^{\mathrm{II}}) - (\sin\psi_{sn} + \lambda\mu_s^{\mathrm{II}}\cos\psi_{sn})\sum_{n=1}^{N}[k_{sp}^{\mathrm{II}} f(\delta_{sn}^{\mathrm{II}}) + c_{sp}^{\mathrm{II}}\dot{\delta}_{sn}^{\mathrm{II}}] + k_{sx}^{\mathrm{II}} x_s^{\mathrm{II}} + c_{sx}^{\mathrm{II}}\dot{x}_s^{\mathrm{II}} = 0 \\ m_s^{\mathrm{II}}(\ddot{y}_s^{\mathrm{II}} + 2\omega_c \dot{x}_s^{\mathrm{II}} - \omega_c^2 y_s^{\mathrm{II}}) + (\cos\psi_{sn} - \lambda\mu_s^{\mathrm{II}}\sin\psi_{sn})\sum_{n=1}^{N}[k_{sp}^{\mathrm{II}} f(\delta_{sn}^{\mathrm{II}}) + c_{sp}^{\mathrm{II}}\dot{\delta}_{sn}^{\mathrm{II}}] + k_{sy}^{\mathrm{II}} y_s^{\mathrm{II}} + c_{sy}^{\mathrm{II}}\dot{y}_s^{\mathrm{II}} = 0 \\ \dfrac{I_s^{\mathrm{II}}}{(r_s^{\mathrm{II}})^2}\ddot{u}_s^{\mathrm{II}} + (1 - \lambda\mu_s^{\mathrm{II}}\dfrac{l_{sn}^{\mathrm{II}}}{r_s^{\mathrm{II}}})\sum_{n=1}^{N}[k_{sp}^{\mathrm{II}} f(\delta_{sn}^{\mathrm{II}}) + c_{sp}^{\mathrm{II}}\dot{\delta}_{sn}^{\mathrm{II}}] + k_{su}^{\mathrm{II}} u_s^{\mathrm{II}} + c_{su}^{\mathrm{II}}\dot{u}_s^{\mathrm{II}} + \dfrac{k_{csu}}{r_c^{\mathrm{II}}}(\dfrac{u_s^{\mathrm{II}}}{r_s^{\mathrm{II}}} - \dfrac{u_c^{\mathrm{I}}}{r_c^{\mathrm{I}}}) + \dfrac{c_{csu}}{r_s^{\mathrm{II}}}(\dfrac{\dot{u}_s^{\mathrm{II}}}{r_s^{\mathrm{II}}} - \dfrac{\dot{u}_c^{\mathrm{I}}}{r_c^{\mathrm{I}}}) = \dfrac{T_s^{\mathrm{II}}}{r_s^{\mathrm{II}}} \end{cases} \tag{2}$$

2) Figure 3b depicts that the ring is subjected to the meshing force from the planets and the supporting. Equations of ring of the FPG are similar with SPG. Only replace I with II, thus, Equations can be obtained.

$$\begin{cases} m_r^{\mathrm{I}}(\ddot{x}_r^{\mathrm{I}} - 2\omega_c \dot{y}_r^{\mathrm{I}} - \omega_c^2 x_r^{\mathrm{I}}) - (\sin\psi_{rn} + \lambda\mu_r^{\mathrm{I}}\cos\psi_{rn})\sum_{n=1}^{N}[k_{rp}^{\mathrm{I}} f(\delta_{rn}^{\mathrm{I}}) + c_{rp}^{\mathrm{I}}\dot{\delta}_{rn}^{\mathrm{I}}] + k_{rx}^{\mathrm{I}} x_r^{\mathrm{I}} + c_{rx}^{\mathrm{I}}\dot{x}_r^{\mathrm{I}} = 0 \\ m_r^{\mathrm{I}}(\ddot{y}_r^{\mathrm{I}} + 2\omega_c \dot{x}_r^{\mathrm{I}} - \omega_c^2 y_r^{\mathrm{I}}) + (\cos\psi_{rn} - \lambda\mu_r^{\mathrm{I}}\sin\psi_{rn})\sum_{n=1}^{N}[k_{rp}^{\mathrm{I}} f(\delta_{rn}^{\mathrm{I}}) + c_{rp}^{\mathrm{I}}\dot{\delta}_{rn}^{\mathrm{I}}] + k_{ry}^{\mathrm{I}} y_r^{\mathrm{I}} + c_{ry}^{\mathrm{I}}\dot{y}_r^{\mathrm{I}} = 0 \\ \frac{I_r^{\mathrm{I}}}{(r_r^{\mathrm{I}})^2}\ddot{u}_r^{\mathrm{I}} + (1 - \lambda\mu_r^{\mathrm{I}}\frac{l_{rn}^{\mathrm{I}}}{r_r^{\mathrm{I}}})\sum_{n=1}^{N}[k_{rp}^{\mathrm{I}} f(\delta_{rn}^{\mathrm{I}}) + c_{rp}^{\mathrm{I}}\dot{\delta}_{rn}^{\mathrm{I}}] + k_{ru}^{\mathrm{I}} u_r^{\mathrm{I}} + c_{ru}^{\mathrm{I}}\dot{u}_r^{\mathrm{I}} = 0 \end{cases} \tag{3}$$

3) Figure 3c clearly depicts that carrier is subjected to elastic forces from the elastic deformation of planets bearing, and also supported by the bearing. Equations of carrier of FPG and SPG can be written as

$$\begin{cases} m_c^{\mathrm{I}}(\ddot{x}_c^{\mathrm{I}} - 2\omega_c \dot{y}_c^{\mathrm{I}} - \omega_c^2 x_c^{\mathrm{I}}) + \sum_{n=1}^{N}(k_{pn}^{\mathrm{I}}\delta_{cnx}^{\mathrm{I}} + \mathrm{c}_{pn}^{\mathrm{I}}\dot{\delta}_{cnx}^{\mathrm{I}}) + k_{cx}^{\mathrm{I}} x_c^{\mathrm{I}} + c_{cx}^{\mathrm{I}}\dot{x}_c^{\mathrm{I}} = 0 \\ m_c^{\mathrm{I}}(\ddot{y}_c^{\mathrm{I}} + 2\omega_c \dot{x}_c^{\mathrm{I}} - \omega_c^2 y_c^{\mathrm{I}}) + \sum_{n=1}^{N}(k_{pn}^{\mathrm{I}}\delta_{cny}^{\mathrm{I}} + \mathrm{c}_{pn}^{\mathrm{I}}\dot{\delta}_{cny}^{\mathrm{I}}) + k_{cx}^{\mathrm{I}} y_c^{\mathrm{I}} + c_{cx}^{\mathrm{I}}\dot{y}_c^{\mathrm{I}} = 0 \\ \frac{I_c^{\mathrm{I}}}{(r_c^{\mathrm{I}})^2}\ddot{u}_c + \sum_{n=1}^{N}(k_{pn}^{\mathrm{I}}\delta_{cnu}^{\mathrm{I}} + c_{pn}^{\mathrm{I}}\dot{\delta}_{cnu}^{\mathrm{I}}) + k_{cu}^{\mathrm{I}} u_c^{\mathrm{I}} + c_{cu}^{\mathrm{I}}\dot{u}_c^{\mathrm{I}} + \frac{k_{csu}}{r_c^{\mathrm{I}}}(\frac{u_c^{\mathrm{I}}}{r_c^{\mathrm{I}}} - \frac{u_s^{\mathrm{II}}}{r_s^{\mathrm{II}}}) + \frac{c_{csu}}{r_c^{\mathrm{I}}}(\frac{\dot{u}_c^{\mathrm{I}}}{r_c^{\mathrm{I}}} - \frac{\dot{u}_s^{\mathrm{II}}}{r_s^{\mathrm{II}}}) = 0 \end{cases} \tag{4}$$

$$\begin{cases} m_c^{\mathrm{II}}(\ddot{x}_c^{\mathrm{II}} - 2\omega_c \dot{y}_c^{\mathrm{II}} - \omega_c^2 x_c^{\mathrm{II}}) + \sum_{n=1}^{N}(k_{pn}^{\mathrm{II}}\delta_{cnx}^{\mathrm{II}} + \mathrm{c}_{pn}^{\mathrm{II}}\dot{\delta}_{cnx}^{\mathrm{II}}) + k_{cx}^{\mathrm{II}} x_c^{\mathrm{II}} + c_{cx}^{\mathrm{II}}\dot{x}_c^{\mathrm{II}} = 0 \\ m_c^{\mathrm{II}}(\ddot{y}_c^{\mathrm{II}} + 2\omega_c \dot{x}_c^{\mathrm{II}} - \omega_c^2 y_c^{\mathrm{II}}) + \sum_{n=1}^{N}(k_{pn}^{\mathrm{II}}\delta_{cny}^{\mathrm{II}} + \mathrm{c}_{pn}^{\mathrm{II}}\dot{\delta}_{cny}^{\mathrm{II}}) + k_{cx}^{\mathrm{II}} y_c^{\mathrm{II}} + c_{cx}^{\mathrm{II}}\dot{y}_c^{\mathrm{II}} = 0 \\ \frac{I_c^{\mathrm{I}}}{(r_c^{\mathrm{I}})^2}\ddot{u}_c + \sum_{n=1}^{N}(k_{pn}^{\mathrm{II}}\delta_{cnu}^{\mathrm{II}} + c_{pn}^{\mathrm{II}}\dot{\delta}_{cnu}^{\mathrm{II}}) + k_{cu}^{\mathrm{II}} u_c^{\mathrm{II}} + c_{cu}^{\mathrm{II}}\dot{u}_c^{\mathrm{II}} = \frac{T_c^{\mathrm{II}}}{r_c^{\mathrm{II}}} \end{cases} \tag{5}$$

4) Planets are subjected to the meshing force from sun and ring. Also, Planets are subjected to the supporting force caused by the bearing. Equations of planets of the FPG and SPG are

$$\begin{cases} m_p^{\mathrm{I}}(\ddot{x}_p^{\mathrm{I}} - 2\omega_c \dot{y}_p^{\mathrm{I}} - \omega_c^2 x_p^{\mathrm{I}}) + (\lambda\mu_s^{\mathrm{I}}\cos\alpha_s - \sin\alpha_s)[k_{sp}^{\mathrm{I}} f(\delta_{sn}^{\mathrm{I}}) + c_{sp}^{\mathrm{I}}\dot{\delta}_{sn}^{\mathrm{I}}] + \\ (\sin\alpha_r + \lambda\mu_r^{\mathrm{I}}\cos\alpha_r)[k_{rp}^{\mathrm{I}} f(\delta_{rn}^{\mathrm{I}}) + c_{rp}^{\mathrm{I}}\dot{\delta}_{rn}^{\mathrm{I}}] - k_{pn}^{\mathrm{I}}\delta_{xn}^{\mathrm{I}} - c_{pn}^{\mathrm{I}}\dot{\delta}_{xn}^{\mathrm{I}} = 0 \\ m_p^{\mathrm{I}}(\ddot{y}_p^{\mathrm{I}} + 2\omega_c \dot{x}_p^{\mathrm{I}} - \omega_c^2 y_p^{\mathrm{I}}) + (-\cos\alpha_s - \lambda\mu_s^{\mathrm{I}}\sin\alpha_s)[k_{sp}^{\mathrm{I}} f(\delta_{sn}^{\mathrm{I}}) + c_{sp}^{\mathrm{I}}\dot{\delta}_{sn}^{\mathrm{I}}] + \\ (\lambda\mu_r^{\mathrm{I}}\sin\alpha_r - \cos\alpha_r)[k_{rp}^{\mathrm{I}} f(\delta_{rn}^{\mathrm{I}}) + c_{rp}^{\mathrm{I}}\dot{\delta}_{rn}^{\mathrm{I}}] - k_{pn}^{\mathrm{I}}\delta_{yn}^{\mathrm{I}} - c_{pn}^{\mathrm{I}}\dot{\delta}_{yn}^{\mathrm{I}} = 0 \\ \frac{I_p^{\mathrm{I}}}{(r_p^{\mathrm{I}})^2}\ddot{u}_p^{\mathrm{I}} + (1 - \lambda\mu_s^{\mathrm{I}}\frac{l_{sn}^{\mathrm{I}}}{r_s^{\mathrm{I}}})[k_{sp}^{\mathrm{I}} f(\delta_{sn}^{\mathrm{I}}) + c_{sp}^{\mathrm{I}}\dot{\delta}_{sn}^{\mathrm{I}}] + (\lambda\mu_r^{\mathrm{I}}\frac{l_{rn}^{\mathrm{I}}}{r_{rn}^{\mathrm{I}}} - 1)[k_{rp}^{\mathrm{I}} f(\delta_{rn}^{\mathrm{I}}) + c_{rn}^{\mathrm{I}}\dot{\delta}_{rn}^{\mathrm{I}}] = 0 \end{cases} \tag{6}$$

Substituting δ_{sn}、δ_{rn}、δ_{cnx}、δ_{cny}、δ_{cnu}、δ_{xn}、δ_{yn} [2] into equations (1-6), dynamics equation of discrete model of TPG are

$$\mathbf{M\ddot{q}} + (\omega_c\mathbf{G} + \mathbf{C_b} + \mathbf{C_m})\mathbf{\dot{q}} + (\mathbf{K_b} + \mathbf{K_m(t)} - \omega_c^2\mathbf{K}_\omega)\mathbf{q} = \mathbf{T} + \mathbf{F(t)} \tag{7}$$

where M is the mass matrix. K_m is the meshing stiffness matrix. Kb is the bearing stiffness matrix. $\mathbf{K}_\omega$ is the centripetal stiffness matrix caused by ω_c of carrier. q is the generalized displacement vector. C_m is the damping matrix. C_b is the supporting matrix. G is the Gyro matrix. T is the force column vector. F(t) is the internal excitation column vector caused by static transmission error.

MODAL ANALYSIS

Assume that the meshing stiffness is constant by ignoring time terms for free vibration. From equation (7), free vibration of this system can be obtained as

$$[M]\begin{bmatrix}\ddot{\mathbf{q}}^{\mathrm{I}} \\ \ddot{\mathbf{q}}^{\mathrm{II}}\end{bmatrix} + \mathbf{(}[K_1] + [K_2]\mathbf{)}\begin{bmatrix}\mathbf{q}^{\mathrm{I}} \\ \mathbf{q}^{\mathrm{II}}\end{bmatrix} = \mathbf{0} \tag{8}$$

where

$$[M] = \begin{bmatrix}\mathbf{M}^{\mathrm{I}} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}^{\mathrm{II}}\end{bmatrix} \qquad [K_1] = \begin{bmatrix}\mathbf{K}^{\mathrm{I}} & \mathbf{0} \\ \mathbf{0} & \mathbf{K}^{\mathrm{II}}\end{bmatrix}$$

$$[K_2] = \begin{bmatrix}\mathbf{K}_b^{\mathrm{I}} & \mathbf{K}_{csu} \\ (\mathbf{K}_{csu})^T & \mathbf{K}_b^{\mathrm{II}}\end{bmatrix}$$

$$\mathbf{q}^{\mathrm{I}}=(x_c^{\mathrm{I}},y_c^{\mathrm{I}},u_c^{\mathrm{I}},x_r^{\mathrm{I}},y_r^{\mathrm{I}},u_c^{\mathrm{I}},x_s^{\mathrm{I}},y_s^{\mathrm{I}},u_s^{\mathrm{I}},x_{p1}^{\mathrm{I}},y_{p1}^{\mathrm{I}},u_{p1}^{\mathrm{I}},\ldots,x_{pn}^{\mathrm{I}},y_{pn}^{\mathrm{I}},u_{pn}^{\mathrm{I}})^{\mathrm{T}}$$

$$\mathbf{q}^{\mathrm{II}}=(x_c^{\mathrm{II}},y_c^{\mathrm{II}},u_c^{\mathrm{II}},x_r^{\mathrm{II}},y_r^{\mathrm{II}},u_r^{\mathrm{II}},x_s^{\mathrm{II}},y_s^{\mathrm{II}},u_s^{\mathrm{II}},x_{p1}^{\mathrm{II}},y_{p1}^{\mathrm{II}},u_{p1}^{\mathrm{II}},\ldots,x_{pn}^{\mathrm{II}},y_{pn}^{\mathrm{II}},u_{pn}^{\mathrm{II}})^{\mathrm{T}}$$

Equation (8) can be regarded as the eigenvalue problem of planetary gears

$$\omega_{\mathrm{i}}\begin{bmatrix}\mathbf{M}^{\mathrm{I}} & \mathbf{0}\\ \mathbf{0} & \mathbf{M}^{\mathrm{II}}\end{bmatrix}\phi_i=\left(\begin{bmatrix}\mathbf{K}^{\mathrm{I}} & \mathbf{0}\\ \mathbf{0} & \mathbf{K}^{\mathrm{II}}\end{bmatrix}+\begin{bmatrix}\mathbf{K}_b^{\mathrm{I}} & \mathbf{K}_{csu}\\ (\mathbf{K}_{csu})^T & \mathbf{K}_b^{\mathrm{II}}\end{bmatrix}\right)\phi_i \quad (9)$$

where $\phi_i=[\mathbf{p}_c^{\mathrm{I}},\mathbf{p}_r^{\mathrm{I}},\mathbf{p}_s^{\mathrm{I}},\mathbf{p}_1^{\mathrm{I}},\ldots\mathbf{p}_n^{\mathrm{I}},\mathbf{p}_c^{\mathrm{II}},\mathbf{p}_r^{\mathrm{II}},\mathbf{p}_s^{\mathrm{II}},\mathbf{p}_1^{\mathrm{II}},\ldots\mathbf{p}_n^{\mathrm{II}}]^{\mathrm{T}}$ is vibration mode. ω_{i} is natural frequency and

$\mathbf{p}_j=[x_j,y_j,u_j],\ j=c,r,s,1,2\ldots n$.

Equation (9) is the inherent characteristic equation, which can be rewritten as follows

$$\mathbf{K}_{csu}\mathbf{p}_s^{\mathrm{II}}+(\mathbf{K}_{cb}^{\mathrm{I}}+\sum\mathbf{K}_{c1}^{n}-\omega_i^2\mathbf{M}_c^{\mathrm{I}})\mathbf{p}_c^{\mathrm{I}}+\sum\mathbf{K}_{c2}^{n}\mathbf{p}_n^{\mathrm{I}}=\mathbf{0} \quad (10)$$

$$(\mathbf{K}_{rb}^{\mathrm{I}}+\sum\mathbf{K}_{r1}^{n}-\omega_i^2\mathbf{M}_r^{\mathrm{I}})\mathbf{p}_r^{\mathrm{I}}+\sum\mathbf{K}_{r2}^{n}\mathbf{p}_n^{\mathrm{I}}=\mathbf{0} \quad (11)$$

$$(\mathbf{K}_{sb}^{\mathrm{I}}+\sum\mathbf{K}_{s1}^{n}-\omega_i^2\mathbf{M}_s^{\mathrm{I}})\mathbf{p}_s^{\mathrm{I}}+\sum\mathbf{K}_{s2}^{n}\mathbf{p}_n^{\mathrm{I}}=\mathbf{0} \quad (12)$$

$$(\mathbf{K}_{c2}^{n})^{\mathrm{T}}\mathbf{p}_c^{\mathrm{I}}-(\mathbf{K}_{r2}^{n})^{\mathrm{T}}\mathbf{p}_r^{\mathrm{I}}+(\mathbf{K}_{s2}^{n})^{\mathrm{T}}\mathbf{p}_s^{\mathrm{I}}+(\mathbf{K}_{pp}^{\mathrm{I}}-\omega_i^2\mathbf{M}_{pp}^{\mathrm{I}})\mathbf{p}_n^{\mathrm{I}}=\mathbf{0}\ (n=1,2\ldots N) \quad (13)$$

$$(\mathbf{K}_{cb}^{\mathrm{II}}+\sum\mathbf{K}_{c1}^{n}-\omega_i^2\mathbf{M}_c^{\mathrm{II}})\mathbf{p}_c^{\mathrm{II}}+\sum\mathbf{K}_{c2}^{n}\mathbf{p}_n^{\mathrm{II}}=\mathbf{0} \quad (14)$$

$$(\mathbf{K}_{rb}^{\mathrm{II}}+\sum\mathbf{K}_{r1}^{n}-\omega_i^2\mathbf{M}_r^{\mathrm{II}})\mathbf{p}_r^{\mathrm{II}}+\sum\mathbf{K}_{r2}^{n}\mathbf{p}_n^{\mathrm{II}}=\mathbf{0} \quad (15)$$

$$\mathbf{K}_{csu}\mathbf{p}_c^{\mathrm{I}}+(\mathbf{K}_{sb}^{\mathrm{II}}+\sum\mathbf{K}_{s1}^{n}-\omega_i^2\mathbf{M}_s^{\mathrm{II}})\mathbf{p}_s^{\mathrm{II}}+\sum\mathbf{K}_{s2}^{n}\mathbf{p}_n^{\mathrm{II}}=\mathbf{0} \quad (16)$$

$$(\mathbf{K}_{c2}^{n})^{\mathrm{T}}\mathbf{p}_c^{\mathrm{II}}-(\mathbf{K}_{r2}^{n})^{\mathrm{T}}\mathbf{p}_r^{\mathrm{II}}+(\mathbf{K}_{s2}^{n})^{\mathrm{T}}\mathbf{p}_s^{\mathrm{II}}+(\mathbf{K}_{pp}^{\mathrm{II}}-\omega_i^2\mathbf{M}_{pp}^{\mathrm{II}})\mathbf{p}_n^{\mathrm{II}}=\mathbf{0}\ (n=1,2\ldots N) \quad (17)$$

Equation (9) is the characteristic equation of TPG. By solving the eigenvalues of this equation, natural frequencies and vibration modes can be obtained. Meshing stiffness between sun and planets, planets and ring are constant. The bearing stiffness of TPG is identical. The angle among sun, planet and ring are identical. For the meshing between sun and planets, planets and ring, the value of integrated curvature radius and base circle radius can be assumed to be $l_{rn}/r_r=l_{\mathrm{sn}}/r_s=0.2$. Structures parameters of TPG are given, as shown in Table 1.

Table 1 - Structures parameters of TPG.

Mass(kg)	$m_c^{\mathbf{I}}=8.346$, $m_r^{\mathbf{I}}=9.423$, $m_s^{\mathbf{I}}=0.79$, $m_p^{\mathbf{I}}=0.62$, $m_c^{\mathbf{II}}=34.457$ $m_r^{\mathbf{II}}=25$, $m_s^{\mathbf{II}}=3.765$, $m_p^{\mathbf{II}}=1.36$
I/r^2(kg)	$I_c^{\mathbf{I}}/r_c^{\mathbf{I}}=8.1055$, $I_r^{\mathbf{I}}/r_r^{\mathbf{I}}=16.8975$, $I_s^{\mathbf{I}}/r_s^{\mathbf{I}}=0.3472$, $I_p^{\mathbf{I}}/r_p^{\mathbf{I}}=0.5714$, $I_c^{\mathbf{II}}/r_c^{\mathbf{II}}=24.135$, $I_r^{\mathbf{II}}/r_r^{\mathbf{II}}=47.08$ $I_s^{\mathbf{II}}/r_s^{\mathbf{II}}=1.55$, $I_p^{\mathbf{II}}/r_p^{\mathbf{II}}=1.12$
Teeth	$\mathrm{Z}_r^{\mathbf{I}}=115$, $\mathrm{Z}_s^{\mathbf{I}}=29$, $\mathrm{Z}_p^{\mathbf{I}}=43$, $\mathrm{Z}_r^{\mathbf{II}}=85$, $\mathrm{Z}_s^{\mathbf{II}}=29$, $\mathrm{Z}_p^{\mathbf{II}}=28$
Angle	$\alpha_{\mathrm{s}}^{\mathbf{I}}=\alpha_{\mathrm{r}}^{\mathbf{I}}=20^{\circ}$, $\alpha_{\mathrm{s}}^{\mathbf{II}}=\alpha_{\mathrm{r}}^{\mathbf{II}}=20^{\circ}$
Modulus	$m^{\mathbf{I}}=1.75$, $m^{\mathbf{II}}=3$
Meshing stiffness (N/m)	$k_{sp}^{\mathbf{I}}=k_{rp}^{\mathbf{I}}=9\times10^{8}$, $k_{sp}^{\mathbf{II}}=k_{rp}^{\mathbf{II}}=1\times10^{9}$
Bearing stiffness (N/m)	$k_p^{\mathbf{I}}=k_r^{\mathbf{I}}=k_s^{\mathbf{I}}=10^{8}$, $k_p^{\mathbf{II}}=k_r^{\mathbf{II}}=k_s^{\mathbf{II}}=10^{8}$
Torsional stiffness (N/m)	$k_{cu}^{\mathbf{I}}=k_{su}^{\mathbf{I}}=k_{cu}^{\mathbf{II}}=k_{su}^{\mathbf{II}}=0$, $k_{ru}^{\mathbf{I}}=k_{ru}^{\mathbf{II}}=10^{9}$

Substituting these parameters into equation (9), free vibration considering friction is obtained in Table 2 with the number of first-stage planets $N^{\mathrm{I}}=4$ and the number of second stage planets $N^{\mathrm{II}}=3$, $N^{\mathrm{II}}=4$ and $N^{\mathrm{II}}=5$, respectively. Table 2 shows that TPG system can be divided into three vibration modes: first-stage vibration mode (FVM), second-stage vibration mode (SVM) and coupled vibration mode (CVM). It shows that with the case of $\mu=0$ and $N^{\mathrm{I}}=4$, the first-stage mode has no change but the second-stage and coupled mode change with N^{II} increasing from 3 to 5. In other words, the FVM 11788.2(2) Hz, 9035.4(2) Hz, 8308.0 Hz, 1915.9 Hz, 771.0(2) Hz, 502.9(2) Hz, 18446.1 Hz, 9424.0 Hz, 2170.6(2) Hz and 1700.4(2) Hz have no change with the number of second-stage planets increasing. For $\mu=0.04$, the above conclusions are still true. It should be noted that (2) denotes that the natural frequency has a double root, which is the lateral vibration modes.

For $N^{\mathrm{I}}=4$ and $N^{\mathrm{II}}=3$, after considering the friction effects, the modes of first-stage planetary gear of 18446.1 Hz、9424.0 Hz、11788.2(2) Hz、9035.4(2)

Hz、8308.0 Hz reduce. Only the mode of 7903.5(2) Hz of the SPGreduces. The rest modal does not change. The coupled modal 16478.8Hz 、 9147.1Hz 、 7215.3Hz 、 2008.0Hz、 1707.1Hz also reduce, which indicts that friction has great influence on the first stage planetary vibration modes and the coupling vibration modes. However, it has little influence on the modes of the SPG. For N^{II}=4 and N^{II}=5, the above rules are basically the same. The friction coupled within meshing stiffness matrix causes the reduction of the whole stiffness matrix, resulting in the reduction of natural frequencies.

For N^{I}=N^{II}=4, the modes are studied when friction is $\mu = 0$ and $\mu = 0.04$. It shows that first stage lateral vibration modal 9035.4(2) Hz, planets vibration modal 9424.0(2) Hz and torsional vibration modal 18446.1 Hz change, which indicates that friction can affect the three types modal of first stage planetary gear. Meanwhile, the lateral modal 8090.1(2) Hz and planets modal 7404.1 Hz of second stage planetary gear change, which indicates that friction can affect the two types modal of second stage planetary gear, and only affect this two modal. Additionally, the couple modal is reduced after considering the friction effects.

As mentioned above, the three kinds of vibration modes of planetary gears have changed at a different degree. It shows that friction has great influence on natural frequencies of planetary gears. Considering friction effects, the modal calculated numerically is closer to the real value, which gives the natural frequencies accurately and has practical engineering significance.

When the planetary gear is drive, gear's meshing force will change, thus, the friction force $f = \mu F_N$ changes at all times. As an internal force, meshing force of gears can be replaced by spring. It can be seen that the friction term is coupled in $\mathbf{K_m}$. Friction affects the meshing stiffness matrix, which is the mechanism of the influence of friction on the natural frequency. Because of the different sliding speed, meshing gears generate the relative sliding friction. Thus, friction only affects the meshing stiffness matrix, and it does not affect the supporting matrix.

Table 2 - The changing of natural frequency with the case of $N^I = 4$, $\mu = 0$ and 0.04.

	N^{II}=3			N^{II}=4			N^{II}=5		
$\mu = 0$	11788.2(2)	9035.4(2)		11788.2(2)	9035.4(2)		11788.2(2)	9035.4(2)	
Modal	2170.6(2)	1700.4(2)		2170.6(2)	1700.4(2)		2170.6(2)	1700.4(2)	
of first	771.0(2)	502.9(2)		771.0(2)	502.9(2)		771.0(2)	502.9(2)	
stage	18446.1	9424.0		18446.1	9424.0		18446.1	9424.0	
(Hz)	8308.0	1915.9		8308.0	1915.9		8308.0	1915.9	
$\mu = 0.04$	11769.6±95.8i	9025.2±13.9i		11769.6±95.8i	9025.2±13.9i		11769.6±95.8i	9025.2±13.9i	
Modal	2170.6(2)	1700.4(2)		2170.6(2)	1700.4(2)		2170.6(2)	1700.4(2)	
of first	771.0(2)	502.9(2)		771.0(2)	502.9(2)		771.0(2)	502.9(2)	
stage	18446.1	9424.0		18446.1	9424.0		18446.1	9424.0	
(Hz)	8308.0	1915.9		8308.0	1915.9		8308.0	1915.9	
$\mu = 0$	7903.5(2)	6560.5(2)		8090.1(2)	6654.2(2)		8281.4(2)	6735.9(2)	
Modal	1357.9(2)	892.9(2)		1377.1(2)	918.6(2)		1396.8(2)	943.2(2)	
of second	407.9(2)	295.1(2)		420.3(2)	293.3(2)		427.2(2)	291.3(2)	
stage				7404.1	6168.1	1301.5	7404.1(2)	6168.1(2)	1301.5(2)
(Hz)									
$\mu = 0.04$	7878.9±22.1i	6559.9±11.9i		8067.0±30.6i	6652.5±13.7i		8259.8±38.8i	6733.1±15.1i	
Modal	1357.5±1.3i	892.6±3.4i		1376.8±1.7i	918.2±4.5i		1396.5±2.2i	942.8±5.6i	
of second	407.9±2.4i	295.1±0.1i		420.3±2.2i	293.2±0.1i		427.2±1.8i	291.2±0.1i	
stage				7376.0	6169.5	1300.9	7376.0(2)	6169.5(2)	1300.9(2)
(Hz)									
$\mu = 0$	16478.8	9147.1	7215.3	17064.9	9147.1	7206.6	17624.7	9147.1	7210.9
Coupled	5938.2	2008.0	1707.1	5859.6	2007.1	1704.0	5784.2	2006.3	1701.3
modal	1303.7	1129.2	781.9	1304.4	1130.0	821.9	1305.1	1131.5	871.2
(Hz)	541.9	0		588.2	0		612.8	0	
$\mu = 0.04$	16462.7	9129.1	7192.1	17044.6	9129.1	7184.4	17600.6	9129.1	7189.4
Coupled	5940.1	2011.5	1700.8	5861.3	2010.5	1697.8	5785.9	2009.6	1695.2
modal	1303.1	1128.6	781.6	1303.8	1129.4	821.6	1304.5	1130.8	870.8
(Hz)	541.8	0		588.2	0		612.8	0	

Ignoring the time terms of meshing stiffness and regarding the mass matrix and stiffness matrix as constant matrix, the system of planetary gear is linear. When the system is subject to an internal constant force, the natural frequency can be obtained from the frequency response curve. Figure 4 shows the frequency response curve of the torsional vibration of sun, and it indicates that 18446.1Hz is the natural frequency of torsional vibration of FPG. 541.9Hz, 1129.2 Hz, 1707.1 Hz, 2008.0 Hz and 9147.1 Hz is the natural frequency of coupled vibration of TPG. Therefore, it concludes that there is coupling modal in the torsional vibration modes, which show that there is strong coupling between the FPG and SPG, which is 'mutual coupling'.

Figure 5 is the lateral vibration modal of planets of FPG. 18446.1Hz is the torsional vibration mode of FPG. 502.9Hz and 2170.6Hz are the lateral vibration modal of FPG. 781.9Hz, 1707.1Hz, 2008Hz and 9147.1Hz are the coupled modes of TPG. It indicates that for the vibration of planets, there are lateral vibration modal, torsional vibration modal and the coupled modal. Thus, it shows that there is strong lateral-torsional coupling of the FPG, which is 'self-coupling'. Also, there is strong coupling between the FPG and the SPG, which is the 'mutual coupling'.

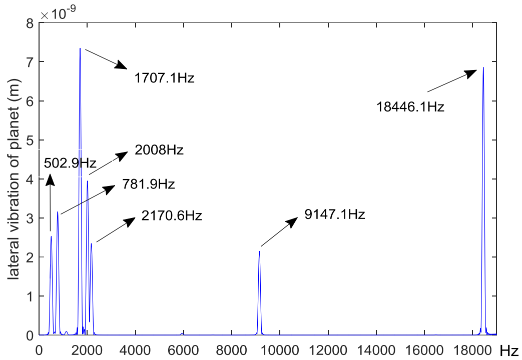

Figure 4. Lateral vibration response of planets.

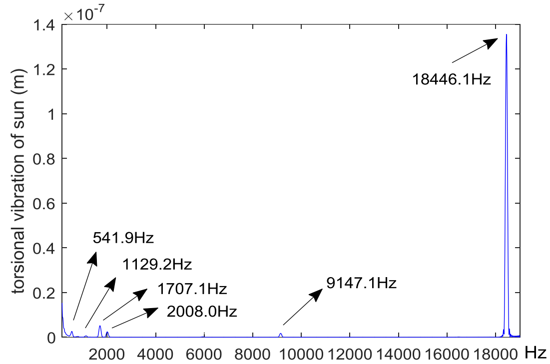

Figure 5. Torsional vibration response of sun.

CONCLUSIONS

This paper focuses on the free vibration of the two-stage planetary gear. The changing of natural frequency of two-stage planetary gear are studied with the case of $N^I=4$ and $N^{II}=3,4,5$. The effect of friction on natural frequency is analyzed. Additionally, the coupling phenomenon is also demonstrated. The research shows that

1) The two-stage planetary gear can be classified into three modes: the first-stage vibration modes, the second-stage vibration modes and the coupled vibration modes. When the number of the planets is determined, the first-stage vibration modes will not change with the increasing of the second-stage planets.

2) There exists strong lateral-torsional coupling relationship for the component of first stage planetary gear, namely, self-coupling. Also, there exists coupling relationship between the first-stage and the second-stage planetary gear, namely, mutual-coupling.

3) As an internal force, friction dissipation the system energy and play a role like damping effect, which is the reason of the appearing of complex mode. The friction is coupled into the stiffness matrix, which makes the whole stiffness asymmetric. Thus, the gear system does not appear two identical roots.

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