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Overview of Structural-Acoustic Modal Analysis under Random Loading

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ABSTRACT

Random excitations can result from various types of non-deterministic loads such as wind loads, terrain loads, and other types of white noise loads. In this paper, an overview is presented of the modal method to obtain the random response of a coupled structural-acoustic system subjected to random excitations. When the structural system is coupled with an enclosed cavity, the structural-acoustic frequency response functions (FRFs) can be obtained using the uncoupled structural modes and the uncoupled acoustic modes, with structural-acoustic coupling as well as modal damping included in the formulation. The random response of the coupled structural-acoustic system is then obtained by summation of the structural-acoustic FRFs with the applied auto- and cross-spectral random loadings at the excitation locations. The theoretical formulation of the coupled structural-acoustic system is described. An example of a rectangular cavity coupled with flexible panels exposed to external random white noise load is presented. The methodology is then applied to an automotive vehicle travelling over a randomly rough road to predict the interior sound pressure response in the vehicle.

INTRODUCTION

In structural-acoustic systems, random loads that may occur in many applications can result in interior noise when a vibrating structure is coupled with an enclosed cavity or in radiated noise from the structure to the surrounding atmosphere. The source of loading excitation can be from structural loads that are random excitations transmitted through the structure or from exterior pressure loads applied to the structure such as from turbulent boundary layer flow excitations or from interior cavity pressure loadings. Depending on the random load type and the particular location of the applied loading, specific structural or acoustic modes may be excited that dominate the response. Also, depending on the structural-acoustic modal coupling effect, coupled structural-acoustic modes may dominate the response.

The classical modal analysis method is well established to the sound pressure response by applying random loads to coupled structural-acoustic system with the modes and frequencies computed from the finite element method [1-3]. The frequency response functions are obtained from the coupled structural-acoustic modal frequency response for prescribed unit loads. For applied random loads, all frequency response function due to load applications and random loads are multiplied and then summed together to get the overall response [1]. Actual external random loads can then be incorporated from test data or simulated results [4]. In the paper, the theoretical formulation of structural-acoustic coupled modal equation and random process is described and illustrated by applications to predict the transmission loss of a shallow cavity coupled with flexible panels exposed to external random white noise excitations and to predict the interior noise in an automotive vehicle travelling over a randomly rough road.

THEORETICAL FORMULATION

Structural-Acoustic Response

The structural-acoustic coupled modal equations-of-motion have been well established and will be briefly reviewed here [1]. For an enclosed cavity coupled with flexible plate type structural walls and rigid walls as shown in Fig.1, the coupled plate-acoustic equations of motion are expressed as,

$$\frac{1}{\rho_o}\nabla^2 p + r\,\dot{p} - \frac{1}{\rho_o c_o^2}\,\ddot{p} = -G + \frac{1}{V}\int_{A_F} \ddot{w}_n dA \qquad (1)$$

$$D\nabla^4 w + r_s\,\dot{w} + \rho_s h\,\ddot{w} = F - \int_{A_F} p\, dA \qquad (2)$$

where in Eq. (1) ρ_o, c_o, r, V, G are the acoustic density, speed of sound, acoustic damping, acoustic enclosure volume and acoustic flow source excitation, and in Eq. (2) $\rho_s, D, r_s, h, A_F, F$ are the structure density, bending rigidity, structural damping, plate thickness, flexible plate area, and structural force excitation. The flexible and rigid wall areas A_F, A_R are as indicated in Fig.1.

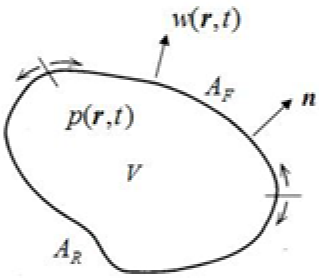

Figure 1. **Enclosed cavity of volume *V* with rigid wall of area A_R and flexible wall of area A_F.**

In terms of a modal expansion, the acoustic pressure and the structural displacement are expressed as,

$$p(\mathbf{r}_a, t) = \sum_n P_n(t)\,\psi_n(\mathbf{r}_a), \quad w(\mathbf{r}_s, t) = \sum_m q_m(t)\,\varphi_m(\mathbf{r}_s) \qquad (3)$$

where φ_m, ψ_n are the uncoupled structural and acoustic mode shapes, $\mathbf{r}_a, \mathbf{r}_s$ represent the acoustic and structural location grid, and q_m, P_n are the coupled structural and acoustic modal response coefficients. Upon substituting Eq. (3) into Eqns. (1) and (2), pre-multiplying ψ_n, φ_m by their transposes, and integrating over acoustic volume and structural surface area, as well as taking advantage of modal orthogonality and the boundary conditions, we have the acoustic modal equation,

$$M_n^A\left(\ddot{P}_n + 2\zeta_n^A\omega_n^A\dot{P}_n + (\omega_n^A)^2 P_n\right) = G_n - \sum_m A_{mn}\ddot{q}_m \qquad (4)$$

that is coupled with the structural modal equation,

$$M_m\left(\ddot{q}_m + 2\zeta_m\omega_m\dot{q}_m + \omega_m^2 q_m\right) = F_m + \sum_n A_{nm}P_n \qquad (5)$$

Here ω_n^A, ω_m are the uncoupled acoustic (rigid wall) and structure mode frequencies, and ζ_n^A, ζ_m are the uncoupled acoustic and structure modal damping coefficients. In Eqns. (4-5), the acoustic modal force G_n, structural modal force F_m, acoustic modal mass M_n^A, structural modal mass M_m, and the structural-acoustic modal coupling coefficient A_{nm} are,

$$G_n = \int_V G(\mathbf{r}_a)\,\psi_n(\mathbf{r}_a)\,dV, \qquad F_m = \frac{1}{A_F}\int_{A_F} F(\mathbf{r}_s)\,\varphi_m(\mathbf{r}_s)\,dA$$

$$M_n^A = \frac{1}{\rho_o c_o^2}\int_V \psi_n^2\,dA, \; M_m = \rho_s h\int_{A_F} \varphi_m^2\,dA \quad A_{nm} \equiv \int_{A_F} \varphi_m\psi_n\,dA \qquad (6)$$

Considering only a single structural force at a particular location $\mathbf{r}_j$ at flexible wall A_F as $F(\mathbf{r}_j) = \delta(\mathbf{r}_j)$, one obtains

$$F_{m,j} = \varphi_m(\mathbf{r}_j), \quad G_n(\mathbf{r}) = 0 \qquad (7)$$

After summation of products of modal response and mode shape, the frequency response functions of acoustic pressure inside the enclosure and the structural walls are,

$$H_{F,j}^p(\mathbf{r}_a, \omega) = \sum_n P_{n,j}(\omega)\,\psi_n(\mathbf{r}_a),$$

$$H_{F,j}^w(\mathbf{r}_s, \omega) = \sum_m q_{m,j}(\omega)\,\varphi_m(\mathbf{r}_s) \qquad (8)$$

Combining all multiple (unit) forces at different locations j, the overall frequency response functions of acoustic pressure inside the enclosure and the structural displacement are,

$$H_F^p(\mathbf{r}_a, \omega) = \sum_j H_{F,j}^p(\mathbf{r}_a, \omega), \; H_F^w(\mathbf{r}_s, \omega) = \sum_j H_{F,j}^w(\mathbf{r}_s, \omega) \qquad (9)$$

A similar procedure can be applied for unit acoustic source excitation $G(\mathbf{r}_{j'}) = \delta(\mathbf{r}_{j'})$ in V to obtain the airborne frequency response functions of the sound pressure and structural displacement as

$$H_G^p(\mathbf{r}, \omega) = \sum_{j'} H_{G,j'}^p(\mathbf{r}, \omega), \quad H_G^w(\mathbf{r}_s, \omega) = \sum_{j'} H_{G,j'}^w(\mathbf{r}_s, \omega) \qquad (10)$$

By summing all structural loading and acoustic source inputs $\{F\}, \{G\}$, the frequency response functions (FRFs) due to structural or acoustic source excitations as in Eqs. (7-10) can be used to predict the structural-acoustic response from all combined loads as,

$$\begin{Bmatrix} w \\ p \end{Bmatrix} = \begin{bmatrix} H_F^w & H_G^w \\ H_F^p & H_G^p \end{bmatrix} \begin{Bmatrix} F \\ G \end{Bmatrix} \qquad (11)$$

Random Modal Analysis

The structural-borne force excitation at several locations with any pair of exciting time dependent forces F_i, F_j has the following relationship over a time interval as [4],

$$R_{F_iF_j} = \lim_{T\to\infty} \frac{1}{T}\int_0^T F_i(t)F_j(t+\tau)d\tau \tag{12}$$

Equation (12) is defined as the force correlation function. When $i = j$, it is defined as auto-correlation function, and when $i \neq j$, it is defined as cross-correlation function. Taking the Fourier transform of correlation function, one obtains the input force power spectrum defined as,

$$S_{F_iF_j}(\omega) = \int_{-\infty}^{\infty} R_{F_iF_j} e^{-i\omega\tau} d\tau \tag{13}$$

The power spectral density (PSD) response can then be evaluated using the standard random analysis procedure [1-3]. For structure borne input forcing spectrum at several input locations $[S_{FF}]$, the output vibration and acoustic response spectrum is

$$\begin{Bmatrix} S_{ww}^F(\omega, \mathbf{r}_a) \\ S_{pp}^F(\omega, \mathbf{r}_s) \end{Bmatrix} = \begin{bmatrix} H_w^F(\mathbf{r}_a, -\omega) \\ H_p^F(\mathbf{r}_s, -\omega) \end{bmatrix}^T [S_{FF}(\omega)] \begin{bmatrix} H_w^F(\mathbf{r}_a, \omega) \\ H_p^F(\mathbf{r}_s, \omega) \end{bmatrix} \tag{14}$$

where $S_{ww}^F(\omega, \mathbf{r}_s)$ is the vibration displacement auto-spectrum PSD response at a structural grid $\mathbf{r}_s$, $S_{pp}^F(\omega, \mathbf{r}_a)$ is the sound pressure auto-spectrum PSD response at an acoustic grid $\mathbf{r}_a$. The above procedure can also be applied in the case of random acoustic source excitation.

For random external pressure excitation to the plate structure, the force excitation input is expressed as

$$\sum_{i=1}^{L}\sum_{i'=1}^{L} S_{F_iF_j}(\omega) \approx \int_A\int_A S_{p_ep_e} dA dA \tag{15}$$

The mean squared response is expressed as the inverse of the Fourier Transform of the auto power spectrum quantities shown in Eq. (12) as [4],

$$\overline{w}^2(\mathbf{r}_s) = 2\int_0^\infty S_{ww}(\mathbf{r}_s, \omega)d\omega, \ \overline{p}^2(\mathbf{r}_a) = 2\int_0^\infty S_{pp}(\mathbf{r}_a, \omega)d\omega \tag{16}$$

And the spatial averaged of the mean squared response is expressed as,

$$\langle \overline{w}^2(\mathbf{r}_s) \rangle = 2\int_0^\infty \langle S_{ww}(\mathbf{r}_s, \omega)\rangle d\omega, \ \langle \overline{p}^2(\mathbf{r}_a) \rangle = 2\int_0^\infty \langle S_{pp}(\mathbf{r}_a, \omega)\rangle d\omega \tag{17}$$

The noise attenuation inside the enclosure is characterized by its *noise reduction* (or *insertion loss*) as,

$$NR = 10 * \log_{10}\left(\frac{S_{p_ep_e}}{S_{pp}}\right) \tag{18}$$

In terms of spatial averaged mean squared response, the noise reduction can also be expressed as [5],

$$NR = 10 * \log_{10}\left(\frac{\overline{p}_e^2}{\langle \overline{p}^2 \rangle}\right) \tag{19}$$

APPLICATIONS

Rectangular Enclosure

Figure 2 shows an enclosure with dimensions 30cm x 15cm x 5cm with flexible wall panels of aluminum with thickness of 0.16cm. The enclosure is exposed to random pressure loading to all panels as shown in Fig. 2(a) through multi speaker excitations. The finite element model was developed to predict structural-acoustic modal response. Figures 2(b) and 2(c) show respectively the finite element model of all flexible walls and the interior cavity. Fig. 2(d) shows the insertion loss in octave frequency band at box center location between FEM result and test data [5].

For the acoustic response, major dominate response is at lowest cavity modes around 600 Hz- 800 Hz. The acoustic pressure loading effect is more evident at the vibration response as shown in Fig. 2(e). As shown, there are significant differences between the uncoupled and coupled surface averaged mean velocity squared vibration response on the wall panels. The panel vibration is mainly dominated by the largest size panels (30 cm x 15 cm). The external pressure loading excitation can only excite volume displacing modes of the panel, but the coupled response shows more panel modal response through coupled interior acoustic mode excitation.

Automotive Vehicle

For a vehicle traveling on a randomly rough road, the tire-road contact force can excite the vehicle body into vibration which in turn will then transmit vibration energy into interior noise as shown in Fig. 3(a). The frequency response functions are computed due to unit force excitation at the front tire and the rear tire with consideration of the phase delay based on the vehicle travelling speed [3]. Figures 3(b) and 3(c) show the finite element model of vehicle structure and interior cavity that are used to develop a coupled structural-acoustic model. The

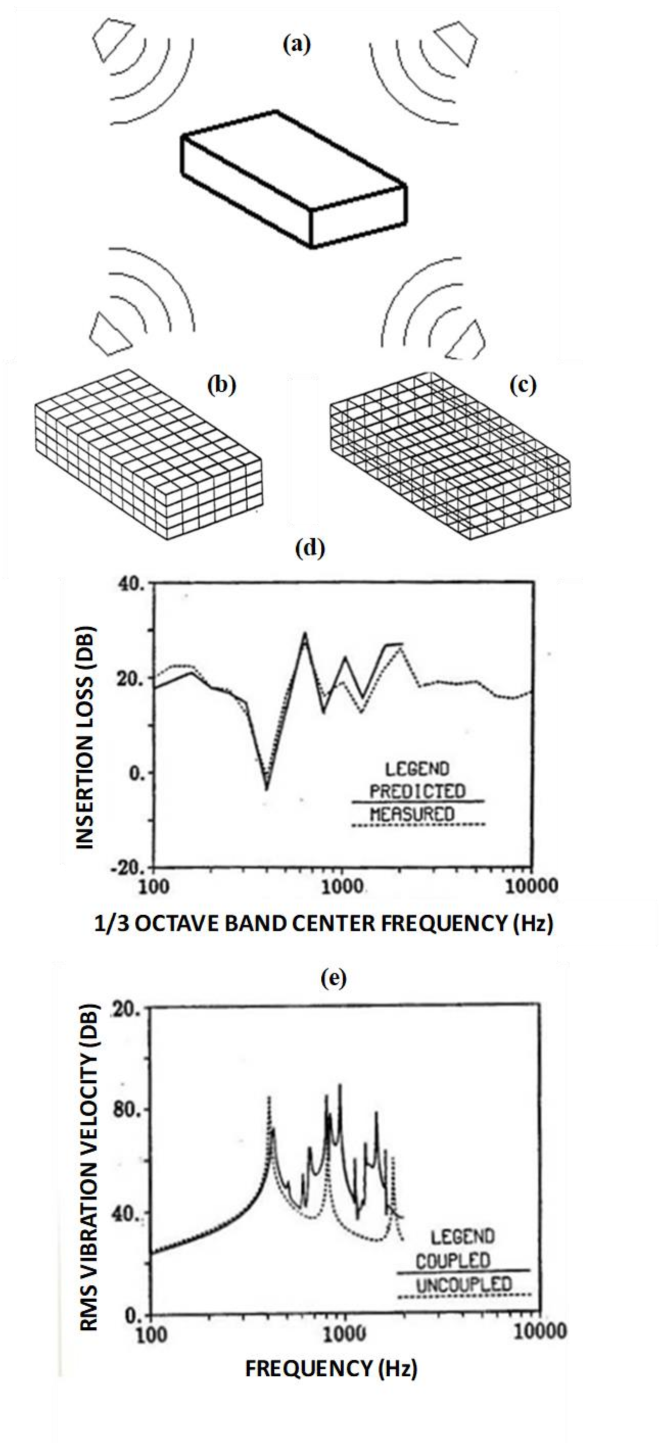

Figure 2. (a) Box in omni-directional speaker excitation, (b) Structural finite element model of box wall panels (c) Acoustic finite element model of box cavity (d) predicted versus measured5 insertion loss at box center, (e) coupled versus uncoupled surface-averaged vibration of 30 cm x 15 cm wall panel [1]

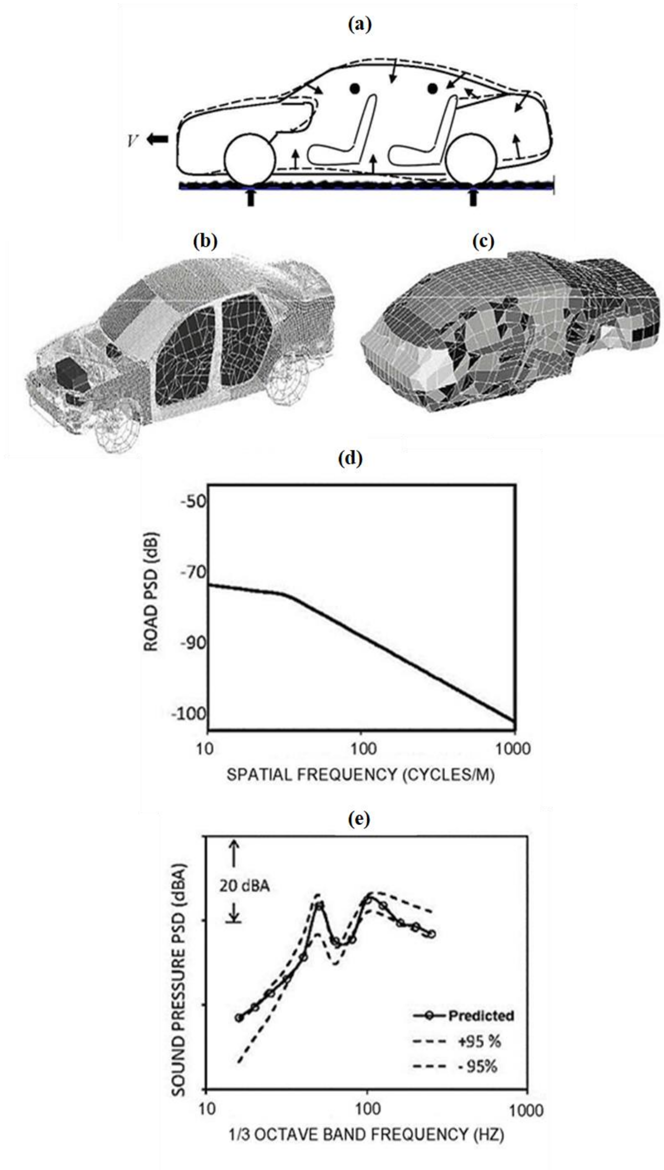

Figure 3. Structural-acoustical analysis to predict sound-pressure PSD response in a vehicle traveling at constant speed V on a randomly rough road: (a) Interior road noise generation. (b) Structural finite-element model of vehicle. (c) Acoustical finite-element model of passenger-trunk compartment. (d) Road profile power spectral density. (e) Predicted A-weighted sound pressure PSD response at front seat occupant ear location versus 95% confidence interval [1].

frequency response functions at both the front tire and the rear tire for each track excitation are then computed. The random road profiles measured for the left and right tracks of a vehicle are considered as stationary random data with their spectral densities (ref. Eq. (13)) of one track as shown in Fig. 3(d) for vehicle speed at 60 KPH.

With the measured input power spectrum and the predicted frequency response functions, Eq. (14) is then applied to predict the sound-pressure PSD response by applying the road-profile PSD function in Fig. 3(d) as excitation at each tire patch at the specified vehicle speed of 60 KPH. Figure 3(e) shows the predicted sound pressure PSD response in the passenger compartment versus the 95% confidence band based on the measured responses in nominally identical vehicles [6].

CONCLUSION

The modal method is presented to obtain the random response of a coupled structural-acoustic system subjected to random excitations. The method is applied to predict the structural-acoustic response for an example of a rectangular enclosure subject to omni-directional random speaker excitation and for the example of an automotive vehicle subject to random road profile excitation. The application of random roads for multiple locations at tire patches considers phase interaction due to a given vehicle speed. Other design inputs such as damping, stiffness treatment can also be easily incorporated in the structural-acoustic modal analysis.

REFERENCES

1. Hambric, S. A. , Sung , S. H. and Nefske, D. J., *Engineering Vibroacoustic Analysis- Method and Applications*, John Wiley & Sons, New York, 2016.
2. S. H. Sung and D. J. Nefske, “Component Mode Synthesis of a Vehicle Structural-Acoustic System Model,” Am. Inst. Aero. Astro. J., 24(6), 1021-1026, (1986).
3. S. H. Sung, D. J. Nefske and D. A. Feldmaier, “A Structural-Acoustic Finite Element Method For Predicting Vehicle Interior Road Noise,” ASME, IMECE 11065, 2009.
4. Newland, D. E., An Introduction to Random Vibrations, Spectral & Wavelet Analysis, 3rd Edition, Longman, London, 1993.
5. I. L. Ver and L. L. Beranek, Noise and Vibration Control Engineering, 2nd Edition, John Wiley & Sons, New York, 2005.
6. J-F Durand, C. Soize and L. Gagliardini, “Structural-Acoustic Modeling of Automotive Vehicles in Presence of Uncertainties and Experimental Identification and Validation,” *J. Acoust. Soc. Am.* **124**(3), 1513-1525, (2008).