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Analysis of Acoustic Radiation Characteristics of an Infinitely Long Half-filled Cylindrical Shell

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ABSTRACT

The acoustic radiation analysis of a fully-submerged infinitely long half-filled cylindrical shell coupling with fluid field is a typical acoustic-structure problem in the infinite domain, the solution of which is currently mainly based on numerical method. The analytic or semi-analytical method is indispensable for the numerical method and the mechanism to reveal the acoustic-structure coupling characteristics. In this paper, an analytic solution is presented that can calculate the acoustic radiation of infinitely long half-filled cylindrical shell. The displacement of the shell, the fluid load and the excitation force are expressed as the combination of trigonometric series and Fourier series, and displacements of the other two directions are removed by orthogonalizing, only the radial displacement is retained. The control equation of the fluid-structure interaction can be obtained from the relationship between the amplitude of fluid load and the amplitude of radial displacement which can be established by orthogonalizing the continuous conditions of the fluid-structure coupled contact surface and the free surface boundary condition. Solving the control equation, the vibration and acoustic radiation of the coupling system can be determined. Compared with the finite element software Comsol, the results of forced

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vibration and underwater radiated noise show that the presented method is accurate and reliable. A new way to solve acoustic-vibration problem with partial coupling of elastic structure and sound field is provided in this study.

1 INTRODUCTION

Due to its excellent geometrical and mechanical properties, the cylindrical shell structure such as oil pipeline, storage tank is widely used in many fields, for example civil engineering, chemical engineering and medical treatment. It seems to be a common problem in the last century, there has been a great deal of literature research on the acoustic-vibration problem of cylindrical shell-fluid coupling system at home and abroad. In this field, most studies are aimed at full fluid filled or full submerged shell (infinite domain), which belongs to the vibration and acoustics of the complete fluid-structure coupling. However, in practical engineering problems, it is common to see the partial coupling of the semi-filled shell and the fluid field. Therefore, it is necessary to research the vibration and acoustic radiation problem of the half-filled shell with full immersion.

Berry et al. [1] are the first to study the vibration characteristics of a shell structure submerged in water completely. They determined the fluctuating pressure on the surface of the fluid cylinder based on the radial acceleration of the point at the cylinder surface, and calculate the natural frequencies in the equation of motion for the shallow cylindrical shells. Bleich et al. [2] studied the vibration of an infinitely long cylindrical shell in an external fluid. They provided a method to determine the vibrated frequencies of infinitely long thin cylindrical shells in an acoustic medium. Expressions are obtained for the displacements of the shell and for the pressures in the medium in the case of forced vibrations due to sinusoidally distributed radial forces. Warburton [3] and Au-Yang [4] studied the effect of both external and internal fluid on shell vibrations. In order to solve the vibration problem of partial liquid filling of the shell, Amabili [5] provided two approximate methods, one is to approximately replace the free surface by the fan boundary which is formed by the circle center of the cylinder, but this method is only suitable for the case of smaller immersion angle, and the other is to replace the original boundary with some annular area. But this method is only suitable for the condition that the immersion angle is less than π. It is noteworthy that the first kind of method proposed by Amabili can be generalized to the problem of partial immersion in the shell [6], and it is pointed out that the application range of immersion angle is only from $-\pi/8$ to $\pi/8$. Instead, Ergin [7, 8] studied the natural frequencies of partial filled or partial submerged cylindrical shells based on the Rayleigh-Ritz method. Guo et al. [9,10] established the analytic model of the coupled finite-depth cylindrical shell-fluid vibration based on the mirror principle and the graf addition theorem, and presented a fast and accurate prediction method of the system's far field sound pressure based on Fourier transform and steady phase method. Li et al. [11] analyzed the acoustic radiation characteristics of a fully filled cylindrical shell under half submerged state and the results show that the acoustic radiation spectra of the semi-submerged shell are quite different from the acoustic radiation spectra of the completely submerged shell. Many numerical results reveal that the sound radiation spectrum of the semi-submerged shell obviously differs from that of a full-submerged shell.

In this paper, the vibration and acoustic radiation characteristics of an infinitely long semi-filled cylindrical shell are analyzed based on the wave propagation method. Because the analytical expression of fluid load is difficult to be obtained, it is proposed to study the semi-filled condition from the special case of partial liquid filling, from which getting the analytical expression of fluid load is more easily. Through the continuous condition of the velocity of the junction of the structure and the fluid, the displacement amplitude can be used to express the fluid load. According to the linear relationship between the three-direction amplitude of variation, the control equation can be simplified to a matrix form which is only related to the radial amplitude, so that it is easier to solve the forced response.

2 THEORETICAL ANALYSIS

2.1 Model introduction

In order to study this issue better, it is assumed that the cylindrical shell is infinitely long in the axial direction, and the excitation force is evenly distributed along the axial direction. Therefore, the mathematical physical model in this paper is a typical plane strain model (i.e. two-dimensional model). The thickness of the two-dimensional cylindrical shell is h and the radius, the radial excitation force, the angle of radial excitation force, Young's modulus, Poisson's ratio and the density are R, f_0, θ_0, E, μ, ρ, respectively. The half-filled cylindrical shell is completely submerged in the fluid with a density of ρ_f, and a sound velocity of c_f,whose axis coincides with the free surface, and the density of the fluid in the shell is ρ_f, too. As is shown in Fig.1, a polar coordinate system (r,θ) is established with the center O of the shell as the origin, which sets as the sound field coordinate system. The forward direction of the coordinate system is as shown in the figure, and the angle of the inner sound field θ ranges from 0 to π, and the same of outer sound field is from 0 to 2π.

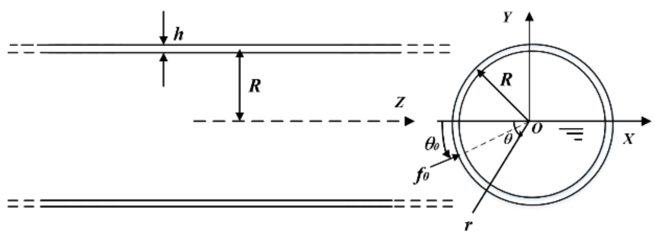


Fig 1. Coordinate figure of the physical model

2.2 Acoustic boundary conditions

The physical model in this paper is a typical sound-structure coupling model. One of the difficulties and focuses of this research work is how to obtain the analytical expression of sound pressure that satisfies all corresponding acoustic boundary conditions.

First, the sound pressure must satisfy the Helmholtz equation:

$$\nabla^2 p + k_f^2 p = 0 \qquad (1)$$

Where $k_f = \omega / c_f$ is the number of sound wave, $\omega = 2\pi f$ is the angular frequency, ∇^2 is the laplace operator.

Second, the sound pressure expression must also satisfy the Sommerfeld radiation condition at infinity:

$$\lim_{R\to\infty}[R(\partial p/\partial R - ik_f p)] = 0 \qquad (2)$$

Where i represents the imaginary number.

According to the physical model established in this paper, the sound field can be divided into the outer sound field and the inner sound field. The sound pressure at the free surface in the shell must meet the sound pressure release conditions (since the research frequency is relatively high, the gravity wave at the free surface can be neglected. Besides, since the air density above the free surface inside the shell is much smaller than that of water, so it can be treated as vacuum.):

$$P_{in}=0 \quad \text{free surface} \quad (3)$$

When the origin of the sound field coordinate system is established on the free liquid surface, the boundary conditions [12] for sound pressure at the free surface can be automatically satisfied by using a sine triangle series, the specific form is as follows:

$$p_{in}(R,\theta)=\sum_{m=1}^{+\infty}P_m(R)\sin(m\theta) \qquad (4)$$

Where m is the ordinal number of sine triangular series, $P_m(R)$ is the corresponding sound pressure amplitude function.

For any point at the free surface, the angle θ=0 or $\theta=\pi$, and put it into (4), the expression of free surface sound pressure can be obtained:

$$\begin{cases} p_{in}(R,\theta{=}0)=\sum_{m=1}^{+\infty}P_m(R)\sin(m\cdot 0)=0 \\ p_{in}(R,\theta{=}\pi)=\sum_{m=1}^{+\infty}P_m(R)\sin(m\pi)=0 \end{cases} \qquad (5)$$

Obviously, such sine trigonometric series can be used to satisfy the sound pressure release conditions of free surface. In addition, the use of trigonometric functions is more conducive to the application of the separation of variables method to solve the Helmholtz equation. From this we can get the analytical expression of the sound pressure amplitude function P_m(R):

$$P_m(R)=B_m J_m^{(1)}(k_f R) \qquad (6)$$

Where $J_m^{(1)}(\ )$ is the m-th order first-type Bessel function , and B_m is the sound pressure amplitude.

Due to the first-type Hankel function automatically satisfies the Sommerfeld radiation conditions in the far field, substituting Equation (6) into Equation (4) , the analytical expression satisfying the acoustic conditions of inner sound pressure can be expressed:

$$p_{in}(R,\theta)=\sum_{m=1}^{+\infty}B_m J_m^{(1)}(k_f R)\sin(m\theta) \qquad (7)$$

Similarly, the sound pressure expression of the outer fluid field is:

$$p_{\text{out}}(R,\theta)=\sum_{n=1}^{+\infty}A_n H_n^{(1)}(k_f R)e^{in\theta} \qquad (8)$$

2.3 Casing equation of motion

After the analytical expression of the sound pressure is obtained, the shell motion equation needs to be established next. The two-dimensional Flügge thin-shell theory is used in this section [5]. Equations are as follows (for the sake of brevity, the harmonic time exp(-iωt) is omitted):

$$[L]\begin{bmatrix} v \\ w \end{bmatrix}=\frac{R_s^{\,2}(1-\mu^2)}{Eh}\begin{bmatrix} 0 \\ f_r-f_p \end{bmatrix} \qquad (9)$$

Where v and w are the tangential and radial displacements of the midplane of the shell, respectively. f_p represents the acoustic load acting on the surface of the cylinder shell, f_0 represents the external excitation load, and [L] is the differential operator matrix of the two-dimensional Flügge thin shell equation , details are as follows:

$$L_{11}=\frac{\partial^2}{\partial\varphi^2}-\frac{\rho R_s^{\,2}(1-\mu^2)}{E}\frac{\partial^2}{\partial t^2},\quad L_{12}=L_{21}=\frac{\partial}{\partial\varphi}$$

$$L_{22}=1+K+2K\frac{\partial^2}{\partial\varphi^2}+K\frac{\partial^4}{\partial\varphi^4}+\frac{\rho R_s^{\,2}(1-\mu^2)}{E}\frac{\partial^2}{\partial t^2}$$

$$K=h^2/12R_s^{\,2}$$

For the cylindrical shell structure, due to the periodicity of the circumferential direction, its displacement and load function can be expanded into the form of Fourier series in the circumferential direction [9]:

$$v = \sum_{n=-\infty}^{+\infty} V_n \exp(in\theta) \tag{10}$$

$$w = \sum_{n=-\infty}^{+\infty} W_n \exp(in\theta) \tag{11}$$

$$f_p = \sum_{n=-\infty}^{+\infty} f_{pn} \exp(in\theta) \tag{12}$$

$$f_0 = \sum_{n=-\infty}^{+\infty} f_{0n} \exp(in\theta) \tag{13}$$

Where V_n and Wn are the amplitudes of the circumferential and radial displacements, respectively, and f_{pn} and f_{0n} represent the amplitudes of the shell surface acoustic load f_p and the excitation force load f_0, respectively, and n is the number of Fourier expansion sequences in the circumferential direction.

The sound pressure inside and outside of the cylindrical shell and the surface vibration of the shell meet the continuous boundary conditions of vibration speed:

$$\begin{cases} \left.\dfrac{\partial p_{out}}{\partial r}\right|_{r=R} = \rho_f \omega^2 w \\ \left.\dfrac{\partial p_{in}}{\partial r}\right|_{r=R} = -\rho_f \omega^2 w \end{cases} \tag{14}$$

Where ρ_f is the density of the fluid, ω is the angular frequency, and w is the radial displacement.

Substituting Equation (7) and Equation (8) into Equation (11) can get the sound pressure amplitude of the inner and outer fluid field:

$$\begin{cases} A_n = \dfrac{\rho_f \omega^2}{k_f H_n^{(1)\prime}(k_f R)} W_n \\ B_m = \dfrac{-2\rho_f \omega^2}{\pi k_f J_m{}'(k_f R)} \bullet \sum\limits_{n=-\infty}^{+\infty} W_n \bullet \xi_{m,n} \end{cases} \tag{15}$$

Where $\xi_{m,n} = \int_0^{\pi} \sin(m\theta) \cdot e^{in\theta} d\theta$

f_{pn} can be obtained by orthogonalizing the expression of different forms of the surface acoustic load f_p of the shell :

$$f_{pn} = \frac{1}{2\pi} \int_0^{2\pi} \left(p_{in} + p_{out} \right)\Big|_{r=R} \cdot e^{-in\theta} d\theta \tag{16}$$

Putting Equation (7), Equation (8) and Equation (10) into Equation (16)：

$$f_{pn} = \frac{\rho_f \omega^2}{k_f} \left[\frac{H_n^{(1)}(k_f R)}{H_n^{(1)\prime}(k_f R)} W_n - \sum_{m=1}^{+\infty} \sum_{a=-\infty}^{+\infty} \frac{J_m(k_f R)}{\pi^2 k_f J_m{}'(k_f R)} \right] \cdot \xi_{m,a} \cdot \xi_{m,-n} \cdot W_a \tag{17}$$

Where $\xi_{m,\mathrm{a}} = \int_0^{\pi} \sin(m\theta) \cdot e^{ia\theta} d\theta$

$$\xi_{m,-n} = \int_0^{\pi} \sin(m\theta) \cdot e^{-in\theta} d\theta$$

Assuming that the excitation force acting on the cylindrical shell is an infinitely long radial linear force distributed along the axial direction, and the excitation force is located at (R,θ_0) of the polar coordinate system, the excitation force can be expressed as follows:

$$f_0 = F_0\delta(\theta-\theta_0) \qquad (18)$$

Where $\delta(\ )$ denotes the Dirac Delta function.

Similarly, Equation (13) and Equation (18) are different forms of the expression of the excitation force f_0. By orthogonalizing, the expression f_{0n} can be obtained:

$$f_{0n} = \frac{F_0\exp(-in\theta_0)}{2\pi} \qquad (19)$$

Then, substituting Equation (10)~(13) into Equation (9) and By orthogonalizing can get the decoupled equation of the shell motion:

$$[T]\begin{bmatrix} V_n \\ W_n \end{bmatrix} = \frac{R_s^2(1-\mu^2)}{Eh}\begin{bmatrix} 0 \\ f_{0n}-f_{pn} \end{bmatrix} \qquad (20)$$

Where the elements of matrix [T] are as follows： $T_{11}=\Omega^2-n^2$ ， $T_{12}=T_{21}=in$ ， $T_{22}=1+K+Kn^4-2Kn^2-\Omega^2$ 。 $\Omega=\omega\sqrt{\rho R_s^2(1-\mu^2)/E}$ is a dimensionless frequency.

Equation (20) can be used to obtain only the control equation related to radial displacement amplitude:

$$W_n = \frac{R_s^2(1-\mu^2)I_n}{Eh}\left(f_{0n}-f_{pn}\right) \qquad (21)$$

Where $I_n = T_{11}/|T|$, I_n isrelated to n, $|T|$ Represents the determinant of the matrix [T].

Obviously, the key to solve the control Equation (21) is to obtain the relationship between the radial displacement amplitude W_n and the shell surface acoustic load amplitude f_{pn}.

Transforming Equation (21):

$$\frac{Eh\cdot W_n}{R^2(1-\mu^2)I_n} + f_{pn} = f_{0n} \qquad (22)$$

From Equation (22):

$$\left[\frac{Eh\cdot W_n}{R^2(1-\mu^2)I_n} + \frac{\rho_f\omega^2}{k_f}\frac{H_n^{(1)}(k_fR)}{H_n^{(1)\prime}(k_fR)}\right]W_n - \sum_{m=1}^{+\infty}\sum_{a=-\infty}^{+\infty}\frac{J_m(k_fR)}{\pi^2k_fJ_m{}'(k_fR)}\cdot\xi_{m,a}\cdot\xi_{m,-n}\cdot W_a = \frac{F_0\exp(-in\theta_0)}{2\pi} \qquad (23)$$

According to Equation (23), B_m, A_n can be solved so that the sound pressure can be calculated.

When the forced vibration is solved in the foregoing, it is the known that excitation force and excitation frequency, and the response (radial displacement amplitude) are found. When solving for free vibration, there is no source of excitation and the natural frequency is the unknown quantity required for the solution. Thus formula (23) can be expressed as:

$$\left[\frac{Eh\cdot W_n}{R^2(1-\mu^2)I_n} + \frac{\rho_f\omega^2}{k_f}\frac{H_n^{(1)}(k_fR)}{H_n^{(1)\prime}(k_fR)}\right]W_n - \sum_{m=1}^{+\infty}\sum_{a=-\infty}^{+\infty}\frac{J_m(k_fR)}{\pi^2k_fJ_m{}'(k_fR)}\cdot\xi_{m,a}\cdot\xi_{m,-n}\cdot W_a = 0 \qquad (24)$$

Matrix calculations are performed on Equation (24) to obtain the matrix eigenvalues, and the natural frequency of free vibration can be obtained.

2.4 Numerical analysis

Model parameters: radius R = 0.18m, thickness h = 0.001m, shell density ρ = 7850kg/m3, Young's modulus E = 206GPa, Poisson's ratio μ = 0.3, fluid density ρ_f =1025kg/m^3, fluid sound velocity C_f=1500m/s.

1.Verification of the accuracy of the method

To illustrate the accuracy of the method for calculating the free vibration problem, the first ten steps natural frequencies of the system are calculated and compared with the finite element software Comsol, which are shown in Tab.1. The finite element model is shown in Fig. 2 and larger version is in Fig.3. The fluid field is centered on the origin of the acoustic coordinates, and the radius is taken as 2m. The perfect matching layer is used to simulate the infinity acoustic boundary. The thickness of the matching layer is taken as 0.05m. The grid contains 10467 domain units and 761 boundary units. Defining the relative error of the natural frequency Error=$|f_1-f_2|/f_2$*100%.

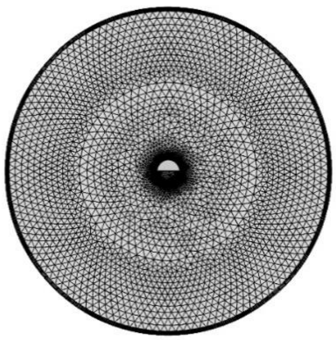


Fig. 2 The finite element model

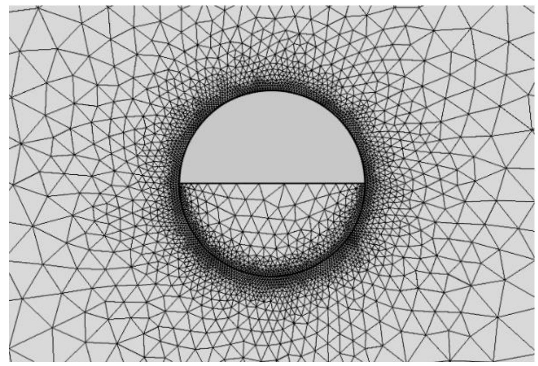


Fig. 3 Larger version of the finite element model

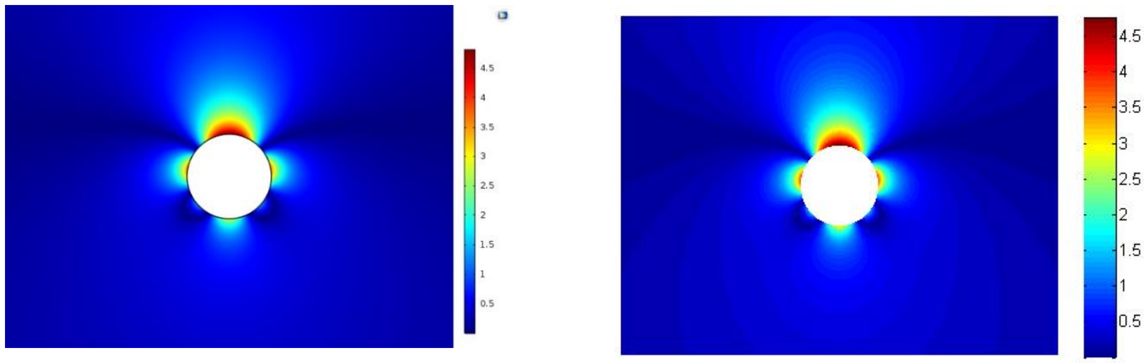
Tab. 1 Comparison of the the first ten order natural frequencies(Hz)

Order	Present(f_1)	FEM (f_2)	*Error*(%)
1	5.35	5.40	0.93
2	5.52	5.55	0.54
3	17.31	17.42	0.63
4	17.61	17.77	0.90
5	37.21	37.37	0.80
6	37.43	37.73	0.79
7	65.64	66.01	0.56
8	65.91	66.47	0.84
9	103.56	104.30	0.71
10	103.81	104.61	0.76

From Table 1, it can be seen that the first ten natural frequencies calculated by this method are in good consistency with Comsol's calculation results, and the maximum relative error does not exceed 1%. This shows that the calculation of the natural frequency using this method is accurate and reliable.

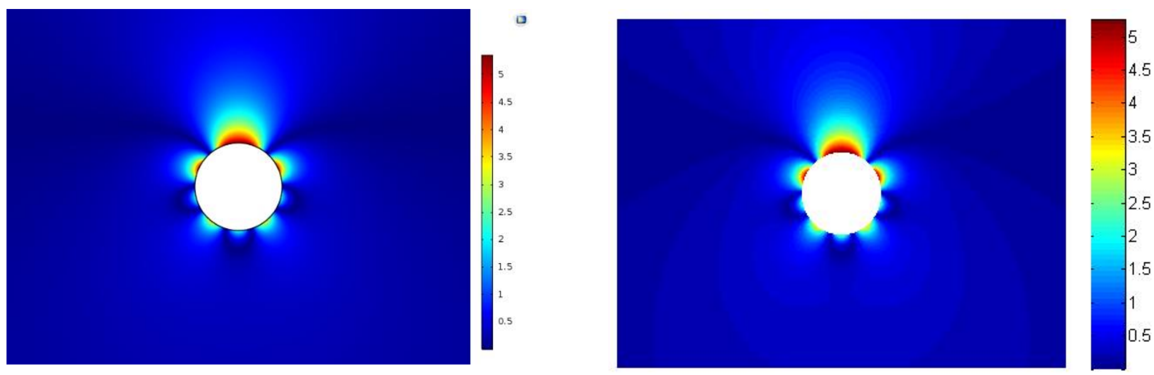
2.Verification of the accuracy of the sound field solution

To further illustrate that the method of calculating the sound field is also accurate, the sound pressure amplitude cloud diagrams with excitation frequencies of 25 Hz, 50 Hz, 100 Hz, and 200 Hz are calculated and compared with Comsol simulation results, which are shown below respectively. Among them, the excitation force amplitude f_0=1N, the excitation angle $\theta_0=3\pi/2$, and the cloud diagrams' size 2m*1.5m.



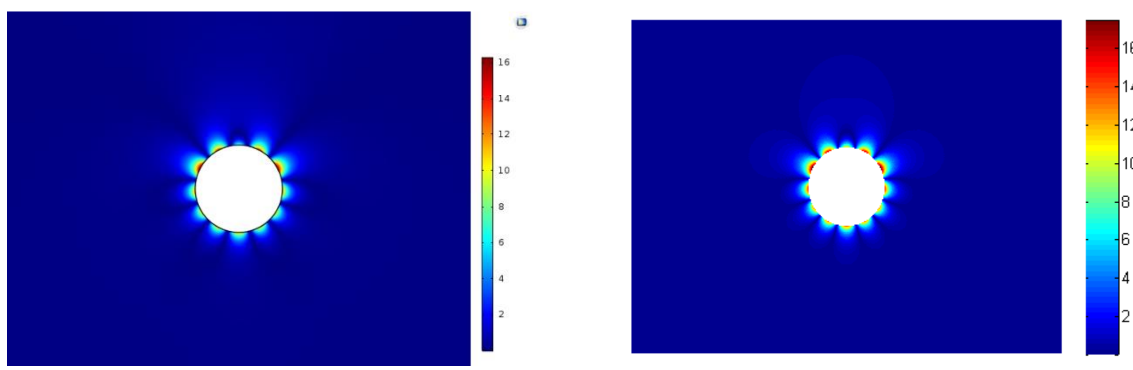
Present method *Comsol*

Fig. 4 Comparison of sound pressure amplitude cloud diagram at excitation frequency 25Hz



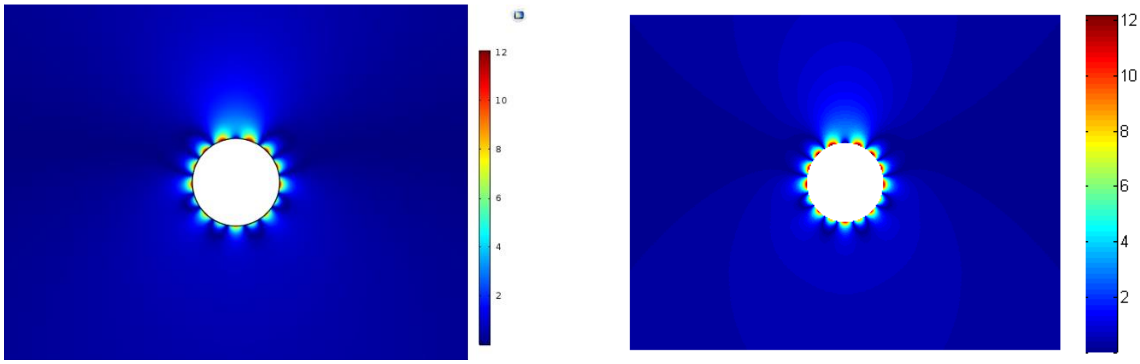
Present method *Comsol*

Fig. 5 Comparison of sound pressure amplitude cloud diagram at excitation frequency 50Hz



Present method *Comsol*

Fig. 6 Comparison of sound pressure amplitude cloud diagram at excitation frequency 100Hz



Present method *Comsol*

Fig. 7 Comparison of sound pressure amplitude cloud diagram at excitation frequency 200Hz

From figure 4 to figure 7, it can be seen that the sound pressure cloud diagram calculated by this method and the results of the Comsol simulation are in good similarity, which can explain this method to calculate the sound field is accurate and reliable.

In addition, it is worth mentioning that the calculation efficiency of this method is also very high. Taking the calculation of the sound pressure cloud diagrams in Figure 4 to Figure 7 as examples, it only takes less than 2 seconds to calculate the accurate and stable results in Matlab.

3. The directivity analysis of far-field sound pressure

Draw the far-field sound pressure directivity plot when the excitation frequency f=50Hz, 200Hz, 1000Hz, 5000Hz, as shown in Figure 8. Where the excitation force amplitude f_0=1N, the excitation position $\theta_0=3\pi/2$. The far field point is taken from the acoustic coordinate system, radius r =1000R, angle θ from 0 to 2 π, the interval is $\pi/180$.

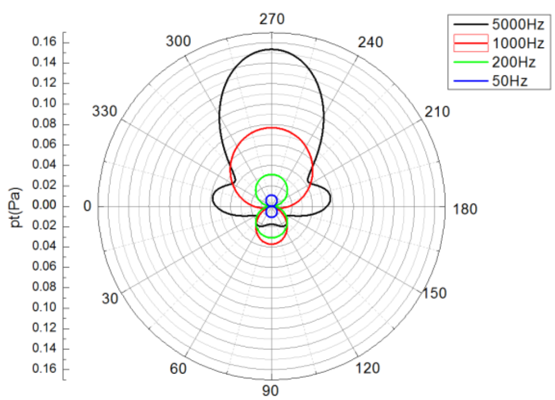


Fig. 8 Far-field sound pressure directivity diagram at different frequencies

From the analysis of Fig.8, it can be obtained that when the excitation force acts on $\theta_0=3\pi/2$, the far-field sound pressure is about the left-right symmetry of the middle profile of the cylindrical shell, and is distributed like a petal. At the same time, when the excitation frequency is higher, the number of petal will increase and the sound pressure value will increase.

3 CONCLUSIONS

Based on the wave propagation method and orthogonality of Trigonal Series and Fourier Series, this paper analyzes the free vibration and acoustic radiation of fully-submerged infinitely long cylindrical shell with semi-filled liquid. Following conclusions can be obtained as:

(1) By analyzing the free vibration, forced vibration and radiated acoustic pressure of partially submerged shells and comparing them with the finite element solution, it can be found the method in this paper is correct. Also the method can be applied to the general situation of the wide range of partially filled.

(2) Based on the orthogonality of series, the amplitude of the three-direction displacement is changed into the amplitude of radial displacement, which simplifies the algorithm and obtains a better result.

(3) Through analyzing of this paper, the sound pressure amplitude of a fully-submerged infinitely long half-filled cylindrical shell increases with the increase of the excitation frequency. The sound pressure in the far field has obvious directivity and symmetry. With the excitation frequency increasing, the number of petal will increase.

An infinitely long submerged half-filled cylindrical shell is taken as an example to study the vibration and acoustic radiation in this paper. In the future, a deeper research will be conducted on the partial filling (immersion) situation, not the semi-filling situation.

4 ACKNOWLEDGEMENTS

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