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SLATCOW METHOD (SPATIAL LAPLACE TRANSFORM FOR COMPLEX WAVENUMBER RECOVERY) FOR FREQUENCY COMPLEX WAVENUMBER DISPERSION RELATION RECOVERY

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ABSTRACT

The study of the dispersion relation of a material is essential for the characterization of the wave propagation. Usually the dispersive regime of a material is activated when its characteristic dimensions are similar to the wavelength of the wave propagating in it. To name but a few, wave guides, sub-wavelength resonators embedded on a propagation medium, or even the structure itself of a natural compound are some examples of systems presenting dispersive behavior. The main feature of these systems is the dependency of its properties with the frequency. Another feature of dispersive systems, related to dispersion, is the (non-geometrical) attenuation. Measuring both the dispersion and the attenuation is of utmost importance in wave physics and for material science, since it provides insightful information on the tested system allowing the characterization of properties (effective properties) of a material (heterogeneous system). The assessment of wave dispersion and attenuation mechanisms can be realized with the complex frequency dependent wave numbers characterization. We proposed the SLaTCoW (Spatial LAplace Transform for COmplexWavenumber recovery) method to overcome many of limitations of these other methods while relying on the usual spatiotemporal signal. We applied SLATCOW on the characterization of viscoelastic skeleton properties of poroelastic materials through guided wave measurement by retrieving the complex wavenumber. We extend the method to take into account the geometrical spreading induce by a point source and we apply the method to the Surface Acoustic Wave at a lossy metasurface characterization.

INTRODUCTION

The characterization of the complex wavenumber is primordial for the understanding of the propagation media. In addition to dispersive effects, such as the existence of band gaps, wave attenuation can be caused by factors such as geometric attenuation or intrinsic material loss (e.g., heat dissipation). In any of these cases, the wave attenuation can be interpreted in terms of complex wavenumbers. Usually the dispersion relation is presented for real wavenumbers with-out accounting for losses. The attenuation of the waves induced by the losses and the geometrical spreading of the energy is additionally taken into account with the imaginary part of the complex wavenumbers. The recovery of complex wavenumbers is of particular interest for the characterization of the viscoelastic properties of materials, in systems such as thin-films, [1, 2] or coated plates, [3] the study of mode interactions, i.e., hybridization or repulsion, [4,5] or the recovery of complex band structures arising from structural periodicity or resonant elements. [4, 6, 7] The dispersion relation is typically

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interpreted in the context of frequency and wavenumber domain information and obtained from discrete spatiotemporal data via discrete Fourier transforms and related methods. [8, 9] However, such techniques typically only supply real wavenumber information (or their magnitudes) from two- dimensional, discrete, spatiotemporal wave propagation information, such as may be obtained from scanned receiver measurements. In this work, we use the SLaTCoW [12, 13] method (Spatial LAplace Transform for COmplex Wavenumber recovery) for the complex wavenumber dispersion relation characterization of porous material and surface acoustic wave at a lossy metasurface with a point source excitation.

The SLaTCoW method

The acronym of the proposed method is SLaTCoW for Spatial LAplace Transform for COmplex Wavenumber recovery. In what follows, we focus on the extraction of complex wavenumber information from 1D discrete spatial field measurements. Denote $\xi_{exp}(x,\omega)$ the wave field that has been measured experimentally along the scan line $x_1 \in [x_0, L]$ with a time dependence of $e^{-i\omega t}$, where ω is the angular frequency. Its spatial Laplace Transform at the complex Laplace wavenumber $s = \kappa_i + i\kappa_r$ is given by:

$$L[\xi_{exp}](s,\omega) = \int_{x_0}^{L} \xi_{exp}(x_1,\omega) e^{-sx_1} dx_1, \tag{1}$$

Then, the set of parameters $\{\zeta\}$ which describes theoretically the wave field propagation is retrieved by minimizing the quadratic difference between $L[\xi_{exp}]$ and the spatial Laplace Transform $L[\xi_{the}]$ of the theoretical ansatz wave field $\xi_{the}(\zeta,\omega)$, that is:

$$\{\zeta\} = argmin(\Delta\{\zeta'\}), \tag{2}$$

where $\Delta\{\zeta\prime\}$ is the quadratic distance between $L[\xi_{exp}]$ and $L[\xi_{the}]$, and it reads:

$$\Delta\{\zeta'\} = \sqrt{\sum_j \mid L[\xi_{exp}](s^j) - L[\xi_{the}(\{\zeta'\})](s^j) \mid^2}, \tag{3}$$

where $s^j = \kappa_i^j + i\kappa_r^j$ being the complex Laplace wavenumbers at which the Laplace Transforms of the fields have been evaluated. Note that for convergence reasons, the minimization is performed for $\kappa_i^j \geq 0$, see Ref. [12, 13] for more details.

Application to porous material mechanical characterization

We first apply the SLaTCoW method to the characterization of the porous materials mechanical parameters. Porous materials are known to be highly dissipative due to viscothermal losses, interaction between the solid and fluid phases, and viscoelasticity of the skeleton. Considering the guided elastic waves propagation in a slab of porous material, Boeckx [10] pave the way on the dispersion relation recovery with the real part of the wavenumbers. The current method is applied to experimentally determine both real and imaginary parts of the wavenumbers, thereby filling the existing gap in attenuation characterization capabilities, and enabling the characterization of viscoelastic parameters of porous materials. The experimental setup is depicted in Fig.1(a). A high porosity ($\phi > 0.95$) melamine foam sample 85 *cm* long, 45 *cm* wide, and 5.5 *cm* thick is glued on a rigid backing. The excitation is provided by a shaker (Bruel and Kjaer type 4810), which is rigidly attached to the sample with a threaded steel rod (20 *mm* in length and 5 *mm* diameter) fixed to the shaker on one side and glued on a 1 *mm* thick aluminum plate of width 10 *cm* and height 1.5 *cm*. This plate is cut at the edge opposite to the threaded steel rod and glued to the porous sample, creating a line source 15 *cm* from the edge of the sample. The resonance of this part was measured to be 4500 Hz. While the measurement could be made at higher frequencies (up to 8000 Hz with a relatively good signal to noise ratio), the results are only shown for frequencies below this limit. The excitations are 300 sine functions equally spaced between 200 Hz and 4095 Hz. The surface skeleton normal displacement $u(x_1,t)$ is acquired at 801 positions along a length of $L = 40$ *cm* with a laser vibrometer (Polytec OFV-503) mounted on a one-dimensional moving stage, which moves the laser along the x-axis after each frequency measurement is accomplished. The vibrometer is connected to a spectral analyzer (Stanford Research Systems SR785), which allows us to directly measure $u(x_1,\omega)$ in the frequency domain. Each measurement is averaged 100 times.

Considering elastic plane waves propagation, the theoretical ansatz function take the form:

$$u_{the}(\{\zeta\},x,\omega) = \sum_n A_n e^{i\phi_n} e^{-s^n x_1} \Pi(x_1 - L) \tag{4}$$

with n the number of plane waves and $\{\zeta\} = \{A_n, \phi_n, s^n\}$ is the set parameters. A_n the amplitude, ϕ_n the phase and $s^n = \kappa_i^n + i\kappa_r^n$ is the complex wavenumber of the n plane wave $\Pi(x_1 - L)$ is the gate function equal to 1 when $x_1 \in [0, L]$ and equal to 0 elsewhere. $\{\zeta\} = argmin(\Delta\{\zeta'\})$ is the result of the minimization problem with

$$\Delta\{\zeta'\} = \sqrt{\sum_n \mid L[u_{exp}](s^n) - L[u_{the}(\{\zeta'\})](s^n) \mid^2}. \tag{5}$$

Figure1(b) depicts the Laplace transform $L[u_{exp}](s^n)]$ at $f = 2453Hz$, where $n = 6$ plane waves can been seen. We choose to focus on the first guided mode, which was the one that was excited most efficiently. Extracting the complex wavenumber for this mode is typically difficult, as several modes are present in its vicinity. The procedure was applied to recover three modes around the first guided mode in order to remove the remaining components of the others. The real and the imaginary wavenumber, κ_r^1 and κ_i^1, respectively, versus frequency are found via the SLaTCoW method and plotted with red circles in Fig.1(c) and Fig.1(d), respectively. The theoretical predictions black line in Figs.1(c) and (d) are obtained using Stroh formalism [11] and a Muller algorithm agree well with the wavenumbers extracted using the SLaTCoW method when the complex shear modulus $N = N_r - iN_i$ is fixed such that $N = 38 - i1.52\ kPa$ (damping factor $N_i/N_r = 0.04$). The poisson ratio is fixed to $\nu = 0.3$. This value is in accordance with the literature [14] This shows the efficiency of the present method to discriminate modes when several are overlapping, and its effectiveness for extracting attenuation parameters. This paves the way for the extraction of new experimental information that enables the development of improved models of viscoelastic porous materials.

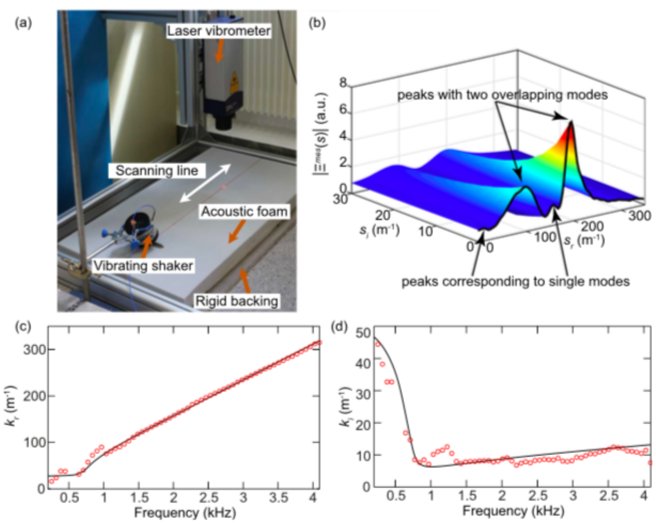

FIGURE 1. (a) Photograph of the experimental setup. (b) Laplace transform $L[\xi_{exp}]$ corresponding to $f = 2453Hz$. Arrows point out peaks, whose main component is a single mode, and peaks, whose main component are two modes overlapping. (c) k_r and (d) attenuation (k_i) of the first guided mode. Red circles depict the results obtained with the SLaTCoW method and solid lines depict the theoretical predictions. Material parameters for the melamine foam ($\phi = 0.989, \rho = 6.1, \alpha_\infty = 1, \sigma = 8060, \Lambda = \Lambda' = 215$).

Application to Surface Acoustic Wave at a lossy metasurface with a point source excitation

Here, the propagation of surface acoustic waves (SAWs) at a metasurface is investigated both theoretically and experimentally to predict and retrieve in a systematic method both the real and imaginary parts of the complex SAW wavenumber in the presence of the viscous and thermal losses. The theoretical model is based on a Boundary Layer (BL) approach using plane wave expansion, and its predictions are used to validate the complex dispersion relation retrieved experimentally. To investigate the SAW propagation at the metasurface, semi-anechoic chamber measurements have been performed. The metasurface consists of 1160 resonators (40 along x_1 and 29 along x_2) drilled periodically with the lattice size $l = 50\ mm \pm 1\ mm$ in a $2m \times 2m$ rigid wooden board (see Fig.2(b)). The speaker has been positioned at the center of the smaller edge ($x_1 = 0 = x_2$) with the center of its membrane at $z_s = 7.5\ cm$ above the metasurface. The system has been excited with a sine-sweep signal over the frequency range $[0.1; 1.6]\ kHz$, in order to focus on the frequency range, wherein the SAW mode theoretically exists according to the BL model (see 2(c)). The pressure field has been measured with the microphone secured to a motorized linear stage. The experimental spectra have been recorded with the Dynamic Signal Analyzer (Stanford Research Systems type SR785) every 0.5 cm along the line $x_2 = 0$ at the distance $x_1 \in [x_0; L] = [5; 180]\ cm$ from the speaker and at $x_3 = 1\ cm$ above the surface. Due to the geometrical spreading of the field from the speaker, the experimental set-up is modelled as a point source above the metasurface admittance. Hence, the theoretical ansatz function of the sound field consists of:

$$p_s(x) = A_0 G_0(x) + A_s G_s(x), \tag{6}$$

where A_0 and A_s are complex amplitudes, G_0 is the Green function for the point source above the rigid surface, and G_s is the perturbation produced by the metasurface admittance, including the SAW:

$$\begin{aligned} G_0 &= e^{ik_0R_1}/(4\pi R_1) + e^{ik_0R_2}/(4\pi R_2), \\ G_S &= -k_0 erfc(-iw) H_0^{(1)}(kr) e^{-b(x_3+z_s)}/4. \end{aligned} \tag{7}$$

Here, the geometrical parameters R_1, R_2, and r are defined in Fig.2(c); $erfc$ is the complementary error function; $H_0^{(1)}$ is the Hankel function of the first kind and order 0; and $w = \sqrt{ik_0R_2 - ikr + b(x_3+z_s)}$ is the numerical distance. Using the pressure $p_s(x)$ as the ansatz field, and hence accounting for the geometrical spreading from the source, with $\{\zeta\} = \{\Re(k_0), \Im(k_0), \Re(k), \Im(k), |A_0|, |A_s|, \arg(A_0), \arg(A_s)\}$ is the set of parameters given by the SLaTCoW method. The experimental results are compared with those from the BL model in

Figs.3(a)-(d), where the complex wavenumbers, the group velocity $v_g = d\omega = d\Re(k)$ and the amplitude ratio A_s/A_0 have been plotted against the frequency. The experimental results are in well agreement with the theoretical model derived for plane waves. It highlights the accuracy of the effective admittance and underlines that the complex dispersion relation is related more to the metasurface admittance and less to the nature of the excitation. The complex dispersion relation theoretically derived has served to validate that experimentally retrieved using the SLaTCoW method, adapted for point source excitation.

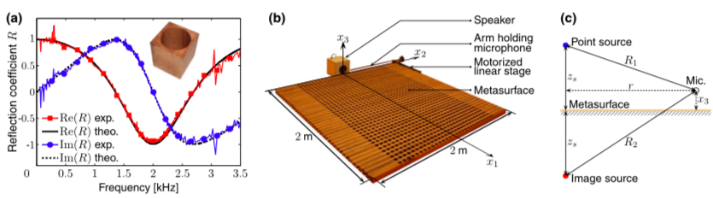

FIGURE 2. (a) Comparison of the real and imaginary parts of the reflection coefficient provided by the impedance tube measurements on a $42\ mm * 42\ mm$ unit cell (see inset) and those provided by the theoretical effective admittance. (b) Experimental set-up for characterisation of SAW propagation at the metasurface prototype. (c) Model of the experimental set-up by a point source and a microphone Mic. above the metasurface admittance.

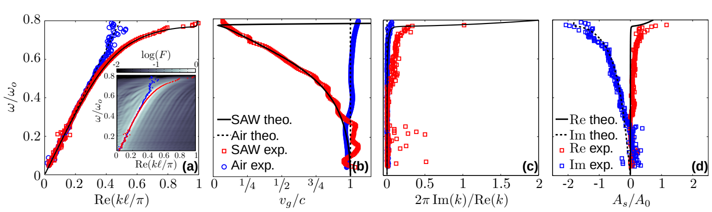

FIGURE 3. Comparison between theoretical (theo.) and experimental (exp.) results. Plotted against the normalized frequency x/xo are: (a) the real part and (c) the imaginary part of the air and SAW wavenumbers; (b) the normalized group velocity; (d) the amplitude ratio between the SAW and the air modes.

Conclusion

A method for the recovery of complex spatial frequency domain information from spatiotemporal data is used. This method, named SLaTCoW (Spatial Laplace Transform for COmplex Wavenumber recovery), is based on a spatial Laplace transform of the measured wave field in the frequency domain, instead of the usual spatial Fourier transform. The Laplace transform, providing information on both the real and imaginary parts of the wavenumbers, is analyzed by the minimization of a correctly chosen ansatz function. We applied SLATCOW on the characterization of viscoelastic skeleton properties of poroelastic materials through guided wave measurement by retrieving the complex wavenumber. We extend the method to take into account the geometrical spreading induce by a point source and we apply the method to the Surface Acoustic Wave at a lossy metasurface characterization. The SLaTCoW method paves the way for new theoretical developments in various fields of physical acoustics such as attenuating and locally resonant materials and underlines the importance of considering the imaginary part of wavenumbers.

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